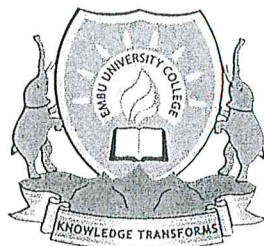# EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2013/2014

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 140: INTRODUCTION TO PROBABILITY AND STATISTICS

DATE: DECEMBER 6, 2013   TIME: 11.00 – 1.00 P.M

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 Marks)

a) Computer chips often contain surface imperfections. For a certain type of computer chip, 9% contains no imperfections, 22% contain 1 imperfection, 26% contain 2 imperfections, 20% contain 3 imperfections, 12% contain 4 imperfections and the remaining 11% contain 5 imperfections. Let Y represent the number of imperfections in a randomly chosen chip.

    (i)   Find $\Pr(Y = y)$ for each possible value of Y

                    (1 mark)

    (ii)   Find the probability mass function of Y

                    (2 marks)

    (iii)   Compute both the expected value of Y and its variance.

                    (5 marks)

    (iv)   Represent the probability mass function graphically

                    (2 marks)

b) The joint p.d.f of X and Y is given by

$$f(x,y) = \begin{cases} 6x^2y, & 0 \leq x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases}$$

(i)   Find $f_1(x)$ and $f_2(y)$ (3 marks)

(ii)   Find μ_x, μ_y, σ_x^2 and σ_y^2 (6 marks)

(iii)   Find ρ, the correlation coefficient between X and Y.   (2 marks)

(iv)   Find $f(x/y)$ and $f(y/x)$ (3 marks)

c) Following are the marks obtained by two students A and B in 10 sets of Examinations.

A's marks	44	80	76	48	52	72	68	56	60	64
B's marks	48	75	54	60	63	69	72	51	57	56

If the consistency of performance is the criterion for awarding a prize, who should get the prize?   (6 marks)

Question 2 (20 marks)

a) Find the mean and the variance of the following distributions:

$$f(x) = \begin{cases} \dfrac{x}{6}, & x = 1, 2, 3 \\ 0, & e/w \end{cases}$$

    (4 marks)

b) Using appropriate illustrations, demonstrate

  (i) Perfect positive correlation   (1 mark)

  (ii) Symmetrical distribution   (1 mark)

c) The probability that a man fishing at a particular place will catch $1, 2, 3, 4$ fish are $0.4, 0.3, 0.2$ and 0.1 respectively. What is the expected number of fish caught?

    (2 mark)

d) The data below gives the marks scored by 90 students in the Statistics class in Embu University College.

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	4	10	A	B	16	C	7	3	1

Given the lower quartile and the mode to be 35 and 44 respectively,

(i)   Find the values of A, B and C   (6 marks)

(ii)   Estimate the median and standard deviation   (6 marks)

Question 3 (20 marks)

a) The data below give the performance of students in the Statistics paper in Embu University College.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students	2	7	21	25	30	35	28	12

(i) Calculate the values of the lower quartile, upper quartile, 3rd deciles and 40th percentile. (8 marks)

(ii) Compute the quartile deviation and comment on it. (2 marks)

b) A die has probability of $\frac{1}{6}$ of coming up 6 when rolled. Let $X = 1$ if the die comes up 6, and $X = 0$ otherwise.

(i) What is the distribution of X (1 mark)

(ii) Find μ_x and σ_x^2 (4 marks)

c) There are five agricultural laborers. They can complete weeding operations on a 100 square meter land in 4, 5, 5, 6, and 7 hours respectively. If these five laborers are employed for weeding in 500 sq. m area, in how many hours will they complete the work? (5 marks)

Question 4 (20 marks)

a) The number of deaths by suicide in a certain city is distributed as a Poisson process with parameter $\lambda = 2$ per day. Let X be the number of deaths by suicide per day. Find:

(i) $P(X = 2)$ and $P(X \geq 3)$ (5 marks)

b) Calculate Karl Pearson's Coefficient of skewness from the given distribution.

Variable	70-80	60-70	50-60	40-50	30-40	20-30	10-20	0-10
Frequency	11	22	30	35	21	11	6	5

Hence comment on the skewness of the distribution (7 marks)

c) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \dfrac{x+y}{21}, & x = 1,2,3; y = 1,2 \\ 0, e/w \end{cases}$$

(i) Find $f_1(x)$ and $f_2(y)$, the marginal probability functions of X and Y respectively. (6 marks)

(ii) Are X and Y independent? (2 marks)

Question 5 (20 marks)

a) Suppose that a random variable X has a Binomial distribution with parameters $n = 8$ and $p = 0.7$, find $P(X \geq 5)$. (4 marks)

b) In an intelligence test administered to 1000 students the average score was 42 and standard deviation 24. Find

(i) The number of students exceeding a score of 50. (3 marks)

(ii) The number of students lying between 30 and 54. (4 marks)

(iii) The value of score exceeded by the top 100 students. (4 marks)

c) A fair coin is tossed 3 times. Let X be the number of heads that are obtained.

(i) Find S (sample space) and use it to find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. (3 marks)

(ii) Find same results as in (i) using binomial distribution. (2 marks)

---END---