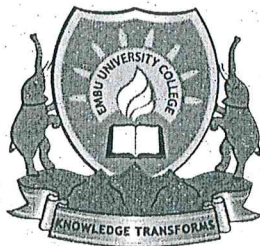


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EMBU UNIVERSITY COLLEGE
(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

SECOND SEMESTER EXAMINATIONS 2013/2014

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF
SCIENCE

SMA 140: INTRODUCTION TO PROBABILITY AND STATISTICS

DATE: APRIL 8, 2014

TIME: 11.00AM - 1.00PM

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) The number of deaths by suicide in a certain city is distributed as a Poisson process with parameter $\lambda = 2$ per day. Let X be the number of deaths by suicide per day. Find:

i.) $P(X = 2)$ and $P(X \geq 3)$ (4 marks)

b) Find the mean and the variance of the following distributions:

i.) $f(x) = \begin{cases} x/6, & x = 1, 2, 3 \\ 0, & \text{elsewhere} \end{cases}$ ($3\frac{1}{2}$ marks)

ii.)
$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases} \quad \left(3\frac{1}{2} \text{ marks}\right)$$

c) Calculate Karl Pearson's Coefficient of skewness from the given distribution.

Variable	70-80	60-70	50-60	40-50	30-40	20-30	10-20	0-10
Frequency	11	22	30	35	21	11	6	5

Hence comment on the skewness of the distribution (7 marks)

d) The joint probability density function of x and y is given by

$$f(x, y) = \begin{cases} \frac{x+y}{21}, & x = 1, 2, 3; y = 1, 2 \\ 0, & \text{elsewhere} \end{cases}$$

i.) Find $f_1(x)$ and $f_1(y)$, the marginal probability functions of x and y respectively and test for independence. (6 marks)

e) A fair coin is tossed 3 times. Let X be the number of heads that are obtained.

i.) Find S (sample space) and use it to find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. (3 marks)

ii.) Find same results as in (i) using binomial distribution. (3 marks)

QUESTION TWO

a) Differentiate between moments and central moments. (2 marks)

b) The joint p.d.f of X and Y is given by

$$f(x, y) = \begin{cases} 6x^2y, & 0 \leq x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

i.) Find $f_1(x)$ and $f_2(y)$ (3 marks)

ii.) Find μ_x , μ_y , σ_x^2 and σ_y^2 (6 marks)

iii.) Find ρ , the correlation coefficient between X and Y . (2 marks)

iv.) Find $f(x/y)$ and $f(y/x)$ (3 marks)

c) Show that the following is a p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases} \quad (4 \text{ marks})$$

QUESTION THREE

a) The following are the results of three University students attained in six tests.

June	11	11	10	10	9	8
Juma	11	10	11	10	10	9
Frank	11	10	10	9	10	10

If on the basis of consistency the University Examination Board awarded June a reward, would the Board be justified? (11 marks)

b) If X has a Binomial distribution, show that $E(X) = np$, $\text{var}(X) = npq$ and

$$m(t) = (q + pe^t)^n. \quad (9 \text{ marks})$$

QUESTION FOUR

a) An incomplete distribution table is given in the table below:

Variable	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	20	?	40	?	25	15

- i.) Given the median value to be 35 and the total frequency to be 170, find the missing frequencies. (3 marks)
- ii.) Calculate the arithmetic mean of the completed table. (2 marks)
- iii.) Estimate the lower and the upper quartiles, hence calculate the semi inter-quartile range. (4 marks)
- iv.) Estimate the 3rd decile and the 20th percentile. (3 marks)
- v.) Estimate the mode of the distribution. (2 marks)

b) The probability distribution function of X is given by

$$f(x) = \begin{cases} \frac{1}{4}, & x = 0, 2 \\ \frac{1}{2}, & x = 1 \\ 0, & \text{elsewhere} \end{cases}$$

i.) Find the moment generating function of X . (2 marks)

ii.) Use (i) above to find the mean and variance of X . (4 marks)

QUESTION FIVE

a) The probability distribution function of X is given by

$$f(x) = \begin{cases} \frac{1}{6}, & x = 1, 2, 3, 4, 5, 6 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i.) μ'_1 , μ'_2 and μ'_3 (5 marks)

ii.) μ_2 and μ_3 (4 marks)

b) In an intelligence test administered to 1000 students the average score was 42 and standard deviation 24. Find

i.) The number of students exceeding a score of 50. (3 marks)

ii.) The number of students lying between 30 and 54. (4 marks)

iii.) The value of score exceeded by the top 100 students. (4 marks)

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