# EMBU UNIVERSITY COLLEGE

**(A Constituent College of the University of Nairobi)**

**2015/2016 ACADEMIC YEAR**

**SECOND SEMESTER EXAMINATION**

**FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS) AND BACHELOR OF SCIENCE (COMPUTER SCIENCE)**

**SMA 108/CSC 113: DISCRETE MATHEMATICS**

**DATE: APRIL 14, 2016** **TIME: 02:00-04:00**

**INSTRUCTIONS:**

**Answer Question ONE and ANY Other TWO Questions.**

## QUESTION ONE

a) State which rule of inference is used in the argument, if it rains today, then we will not have a barbeque today. If we do not have a barbeque today, then we will have a barbeque tomorrow. Therefore if it rains today then we will have a barbeque tomorrow. (2 Marks)

b) Give a direct proof of the theorem, if n is odd, then $n^2$ is odd. (4 Marks)

c) Find the greatest common divisor of ***414*** and ***662*** using the Euclidean algorithm. (4 Marks)

d) Find an integer $0 \leq a < 18$ such that $3958 \equiv a(mod 18)$ (4 Marks)

e) The chairs of an auditorium are to be labeled with a letter and a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently? (2 Marks)

f) Let a function f be defined on a set of real numbers as

$f(x) = \frac{x-2}{x+2}$. Find the inverse of $f(x)$ (3 Marks)

g) If $f(x) = 1 + x + x^2 + \cdots x^n$ ..... and

$g(x) = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n$ ..... Find $f(x)g(x)$ (3 Marks)

h) Eleven books are arranged on a shelf in alphabetical order by author name. In how many ways can one rearrange these books so that no book is in its original position?

(3 Marks)

i) Write the inverse, converse and contrapositive of the conditional statement "if he works hard, then he will pass the examination"

(3 Marks)

j) How many different ways can the letters HIPPOPOTAMUS be written if all the letters are used?

(2 Marks)

## QUESTION TWO

a) Use mathematical induction to show that $1 + 2 + 2^2 + \cdots 2^n = 2^{n+1} - 1$ for all non-negative integers *n* (7 Marks)

b) An urn contains fifteen red numbered balls and ten white numbered balls. A sample of five balls is selected.

i) How many different samples are possible (2 Marks)

ii) How many samples contain all red balls (2 Marks)

iii) How many samples contain three red balls and two white balls (3 Marks)

c) In how many ways is it possible to sit John, Joan, Judy, James, Joseph, Janet and Juliet at a round table if Joseph, Janet and Juliet insist on sitting together?

(2 Marks)

d) Given that U $=\{x: x \text{ } is \text{ } an \text{ } integer, 2 < x \leq 18\}$, $A = \{x: 5 \leq x < 17\}$ and P $= A^c$. List the elements of $A$ *and* $P^c$ (4 Marks)

## QUESTION THREE

a) i) A discrete mathematics class contains 25 students majoring in computer science, 13 students majoring in mathematics and 8 joint mathematics and computer science major. How many students are in this class if every student is majoring in mathematics, computer science or both. (2 Marks)

ii) A total of 1232 students have taken a course in Spanish, 879 in French and 114 in Russian. Further 103 have taken courses in both Spanish and French, 23 in both Spanish and Russian and 14 in both French and Russian. If 2092 students have taken at least one Spanish, French and Russian, how many students have taken a course in all three languages. (4 Marks)

b) Use the truth table to determine whether the given argument form is valid (6 Marks)

$p \vee q$

$p \vee q$

$p \rightarrow r$

$\therefore r$

c) Let f : ℝ→ ℝ and g : ℝ→ ℝ be defined by $f(x) = x+2$, $g(x) = \frac{1}{x^2+1}$

Compute $fog(x)$ and $go(f^{-1}(x))$ (8 Marks)

## QUESTION FOUR

a) Solve the recurrence relation $a_n = 3a_{n-1}\ n \geq 1$ given $a_0 = 1$

(8 Marks)

b) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent where $p,q$ and $r$ are propositions.

(7 Marks)

c) Let A = {1,2} and B = {1,2,3} define a binary relation ***R*** from *A* to *B* as follows.

$R = \{(a, b) \in A \times B | \quad a < b\}$

i) Find the ordered pair in ***R*** (3 Marks)

ii) Find the domain and range of ***R*** (2 Marks)

## QUESTION FIVE

a) Prove that $\sqrt{2}$ is an irrational number (7 Marks)

b) Let$R_1 = \{(1,2), (2,3), (3,4)\}$ and

$R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)(3,4)\}$ be relations from $\{1,2,3)\}$ $to\{1,2,3,4\}$, find

i) $R_1 \cup R_2$ (2 Marks)

ii) $R_1 \cap R_2$ (2 Marks)

iii) $R_2 - R_1$ (2 Marks)

c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = -3x^2 + 2$. Determine whether *f* is one – to - one and onto. (7 Marks)

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