EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF ECONOMICS

SMA 103: CALCULUS I

DATE: APRIL 8, 2016 **TIME: 11:00-13:00**

INSTRUCTIONS: **Answer Question ONE and ANY Other TWO Questions.**

QUESTION ONE

a) Evaluate the following limits:

(i) $\lim_{x\to 2} \frac{x^2-4}{\sqrt{x+7}-3}$ (4 marks)

(ii) $\lim_{x\to \infty} \frac{x+2}{x(x-2)}$ (2 marks)

b) Let a function f be defined by

$$f(x) = \begin{cases} kx^2 + 2x, & \text{if } x < 2 \\ x^3 - kx, & \text{if } x \geq 2 \end{cases}$$

For what value(s) of the constant k is the function f continuous on $(-\infty, \infty)$? (4 marks)

c) Using the definition of derivative, determine $f'(x)$ given that $f(x) = \sqrt{2x+1}$ (6 marks)

d) For each of the following functions, determine $\frac{dy}{dx}$

(i) $y = 2x^5 + 2\sqrt{x} - \frac{3}{\sqrt[3]{x}} + \frac{4}{x^2}$ (2 marks)

(ii) $y = x \ln x$ (2 marks)

(iii) $y = \frac{2x^2-5e^x}{3-\cos x}$ (3 marks)

(iv) $y = \sqrt[5]{2x^2+5x+9}$ (2 marks)

e) If $x^2 + y^2 = 1$, determine $\frac{d^2y}{dx^2}$ in terms of x and y only. (3 marks)

f) In a certain company, the total cost (in Kenya shillings) of producing x items is given by $C(x) = \frac{1}{10}x^3 - 4x^2 + 20x + 5$. Determine the marginal cost at a production level of 100 items. (2 marks)

QUESTION TWO

a) If $x = 2t^2$ and $y = e^t$, determine $\frac{d^2y}{dx^2}$ (simplify your final answer). (5 marks)

b) Determine the values of a and b that make the function f continuous everywhere

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2 \\ ax^2 - bx + 3, & \text{if } 2 \leq x < 3 \\ 2x - a + b, & \text{if } x \geq 3 \end{cases}$$ (6 marks)

c) Determine the production level that will maximize profit for a company whose cost function is given by $C(x) = 16000 + 500x - 1.6x^2 + 0.004x^3$ and demand function is given by $p(x) = 1700 - 7x$ (9 marks)

QUESTION THREE

(a) Determine $\frac{dy}{dx}$ in each of the following:

(i) $y = 5^x$ (4 marks)

(ii) $y = \frac{\ln(2x)}{2-x}$ (4 marks)

(b) Determine an equation of the tangent line to the curve $2x^3 - 9xy + 2y^3 = 0$ at the point (1,2). (4 marks)

(c) The total revenue received from the sale of x units of a certain product is given by the function $R(x) = 12x + 2x^2 + 6$. Determine the following:

(i) The average revenue (2 marks)

(ii) The marginal revenue. What is the interpretation of marginal revenue? (3 marks)

(iii) The marginal revenue at $x = 50$ (1 mark)

(iv) The actual revenue from selling the 51^{st} item (2 marks)

QUESTION FOUR

(a) If $y = \sin^{-1}(2x)$, determine $\frac{dy}{dx}$ (2 marks)

(b) If $y = \frac{1}{x+7e^x}$ determine $\frac{d^2y}{dx^2}$ (4 marks

(c) Given that $z = 2x^3 + 6xy + 5y^3$, determine

(i) $\frac{\partial z}{\partial x}$ (1 mark)

(ii) $\frac{\partial z}{\partial y}$ (1 mark)

(d) Given that $e^{\frac{x}{y}} = x - y$, determine $\frac{dy}{dx}$ (4 marks)

(e) Determine the absolute maximum and absolute minimum values of the function

$f(x) = x^3 - 3x^2 + 1, -\frac{1}{2} \leq x \leq 4$. Sketch the graph of f (8 marks)

QUESTION FIVE

a) For the function $f(x) = e^{-\frac{x^2}{2}}$ determine $f'''(x)$. (3 marks)

b) Evaluate the limit $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5}$ (2 marks)

c) The total revenues (in million shillings) from lease of a building constructed through public-private partnership are approximated by the function $R(x) = \frac{140x^2}{2+x^2}$, where x is the number of years since the opening of the building for leasing.

(i) How fast are the total revenues changing 3 years after the building is opened for leasing? (3 marks)

(ii) What will be the total revenues in the long run? (3 marks)

d) Use Lagrange Multipliers to find the maximum and minimum values of the function $f(x, y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$. (9 marks)