EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2014/2015

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 103: CALCULUS I

DATE: DECEMBER 17, 2014   TIME: 16:00 – 18:00

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Determine the domain of the function $f(x) = \sqrt{x^2 - x - 6}$.   (2 marks)

b) Find $\lim_{x\to 3} \frac{x^2 - 9}{x - 3}$.   (3 marks)

c) Use the Intermediate Value Theorem to show that the polynomial function

$f(x) = 8x^3 + 16x - 9$ has a zero in the interval $[0,1]$.   (4 marks)

d) From first principles, determine the derivative of the function $f(x) = \sqrt{x}$.   (4 marks)

e) Find the equation of the normal to the curve $y = \frac{5}{x^2}$ at the point where $x = 1$.   (4 marks)

f) Differentiate the following functions with respect to x:

   i) $y = \sqrt[3]{(5x - 1)^2}$.   (3 marks)

   ii) $x^2 - y^2 + y = 1$.   (3 marks)

   iii) $y = \tan 2x$ using Quotient rule.   (4 marks)

g) Determine the value of y given $\frac{dy}{dx} = 2x + 1$, $y(0) = 3$.   (3 marks)

QUESTION TWO

a) Evaluate

   i) $\lim_{x \to 1}(\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1})$.   (4 marks)

   ii) $\lim_{x \to 0}\frac{\sin 2x}{\sin 3x}$.   (4 marks)

b) Discuss the continuity of the function $(x) = \frac{x^2-4}{x-2}$.   (3 marks)

c) Given $y = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are differentiable functions of x,

   show that $\frac{dy}{dx} = \frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$.   (6 marks)

d) Determine $\frac{dy}{dx}$ given $x^3 + xy^2 - y^3 = 5$.   (3 marks)

QUESTION THREE

a) Using $y = \sqrt{x}$, estimate the value of $\sqrt{101}$.   (3 marks)

b) Sketch the intersecting graphs of the equations $2x^2 + y^2 = 6$ and $y^2 = 4x$ and show that they are orthogonal.   (6 marks)

c) From the first principles, show that:

   i) $\frac{d}{dx}(sinx) = cosx$.   (4 marks)

   ii) $(\frac{d}{dx}(lnx) = \frac{1}{x}$.   (7 marks)

QUESTION FOUR

a) Given $x = sint$ and $y = cos2t$, show that $\frac{d^2y}{dx^2} + 4 = 0$.   (3 marks)

b) i) Find the equation of the tangent to the curve $y = x^2 - 6x + 5$ at each of the points where it crosses the x-axis.   (6 marks)

ii) Find also the coordinate of the point where the tangents in (b) (i) above meet.

(2 marks)

c) A glass which is in the form of a cone of height $20cm$ and base radius $4cm$ is being filled from a tap at the rate of $25\ ^{cm^3}/_{sec}$ .How fast is the level of water rising at the instant when the height of the water in the glass is $10cm$? (4 marks)

d) Find the area enclosed by the curves $y = x^2 - 4x + 2$ and $y = 2 - x^2$. (5 marks)

QUESTION FIVE

a) Given $y = 2^{3x}$, determine $\frac{dy}{dx}$. (3 marks)

b) If $e^x y = sinx$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$.

(6 marks)

c) Evaluate $\lim_{x \to \infty} \sqrt[3]{\frac{x^2+2}{8x^2-1}}$ (4 marks)

d) A manufacturer wants to design an open box having a square base and a surface area of $108cm^2$ .What is the maximum volume of the box? (7 marks)

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