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**EMBU UNIVERSITY COLLEGE**  
(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2013/2014

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF  
SCIENCE

SMA 101: BASIC MATHEMATICS

**DATE: DECEMBER 6, 2013**

**TIME: 8.30 – 10.30AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

- a) Differentiate the following terms giving relevant examples:
- (i) Relative complement and Absolute complement (3 marks)
  - (ii) Power set and Universal set (2 marks)
  - (iii) Graph and Multigraph. (2 marks)
- b) Given the universal set  $U = \{0,1,2,3,\dots,20\}$ . Let  $A = \{1,2,3,4,5,6,7,8\}$ ,  
 $B = \{1,3,5,7,9,10\}$  and  $C = \{2,4,6,8,10,11\}$ .
- Show that,  $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$  (4 marks)
- c) Verify that the proposition
- I.  $p \vee \sim (p \wedge q)$  is a tautology. (2 marks)
  - II.  $(p \wedge q) \wedge \sim (p \vee q)$  is a contradiction (2 marks)
- d) Prove that  $\sqrt{2}$  is not a rational number. (6 marks)
- e) Express  $4(\cos 135^\circ + j \sin 135^\circ)$  in rectangular form and graph it in the complex plane. (3 marks)

- f) If  $\theta$  is a 2<sup>nd</sup> quadrant angle and  $\sin \theta = \frac{4}{5}$ ; find  $\sin 2\theta$ ,  $\cos 2\theta$  and  $\tan 2\theta$ . (6 marks)

**QUESTION TWO (20 MARKS)**

- a) How many different committees of 4 could be appointed in a club containing 10 members? (1 mark)
- b) Suppose a cancer specialist has four patients: one, two, three, and four. The specialist wants to select three patients for new treatment regime. How can the specialist select three patients out of the four patients? List down the groups. (5 marks)
- c) Find the four fourth roots of  $-16$  (6 marks)
- d) A researcher interviews 150 students in science: 70 were Physics students; 50 were registered in Chemistry; 90 were Biology students; 30 were registered in Physics and Chemistry; 20 registered in Chemistry and Biology; 30 registered in Physics and Biology and 10 registered in Chemistry, Biology and Physics.

Using a Venn diagram, find the following:

- (i) The number of students registered in 2 courses only (2 marks)
- (ii) The number of students registered in only one course (2 marks)
- (iii) The number of students registered in none of the three courses (2 marks)
- (iv) The number of students registered in at least two courses (2 marks)

**QUESTION THREE (20 MARKS)**

- a) Draw a graph of each of the following directed graphs  $G$  where  $V(G) = \{A, B, C, D, E\}$  and
- (i)  $E(G) = \{[A, B], [A, C], [B, C], [B, D], [C, C], [D, B]\}$  (3 marks)
- (ii)  $E(G) = \{[A, D], [B, C], [C, E], [D, B], [D, D], [D, E], [E, A]\}$  (3 marks)
- b) Sketch the graph of  $h(x) = x^3 - 3x^2 - x + 3$  given  $-2 \leq x \leq 4$  (4 marks)
- (i) How many real roots do  $h$  have? (2 marks)
- (ii) Find  $h^{-1}(A)$  where  $A = [-15, 15]$  (2 marks)
- c) Let  $S$  be the relation from  $A = \{Ellen, Stephanie, Audrey, Jane\}$  to  $B = \{yes, no\}$  defined by  $S = \{(Ellen, no), (Stephanie, yes), (Audrey, yes), (Jane, no)\}$
- (i) Find the matrix  $M$  representing the relation  $S$  (2 marks)

(ii) Find the inverse  $S^{-1}$  of the relation  $S$  and the matrix  $N$  representing  $S^{-1}$  (4 marks)

**QUESTION FOUR (20 MARKS)**

a) Prove that disjunction distributes over conjunction:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \quad (4 \text{ marks})$$

b) Divide  $3(\cos 2\theta^\circ + j \sin 2\theta^\circ)$  by  $12(\cos 8\theta^\circ + j \sin 8\theta^\circ)$  and express the result in rectangular form. (6 marks)

c) Show that the number of permutations of a set of  $n$  different objects taken  $r$  at a time, without repetition, is

$$P_{(n,r)} = \frac{n!}{(n-r)!} \quad (6 \text{ marks})$$

d) Find the number of permutations of the letters of the word TENNESSEE. (4 marks)

**QUESTION FIVE (20 MARKS)**

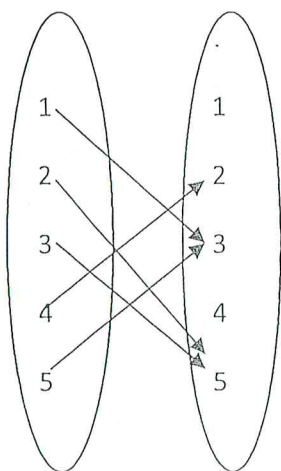
a) Given the universal set  $U = \{0,1,2,3, \dots, 20\}$ . Let  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,  
 $B = \{1, 3, 5, 7, 9, 10\}$  and  $C = \{2, 4, 6, 8, 10, 11\}$ .

(i) Find  $A^c \cup (B \cap C)^c$ , and (3 marks)

(ii)  $(A \setminus C) \cup (B \cap C)^c$  (3 marks)

b) Express  $\cos^2 x$  as an expression involving only functions of  $x$  raised to the first power. (5 marks)

c) Consider the set  $A = \{1,2,3,4,5\}$  and the function  $f: A \rightarrow A$  defined by the figure below.



Find:

(i) The image of each of the element of  $A$  (1 mark)

(ii) The image  $f(A)$  of the function  $f$  (1 mark)

(iii) The graph of  $f$  (1 marks)

(iv)  $f(S)$  where  $S = \{1,3,5\}$  (2 marks)

(v)  $f^{-1}(T)$  where  $T = \{1,2\}$  (2 marks)

(vi)  $f^{-1}(3)$  (2 marks)

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