



MASENO UNIVERSITY
UNIVERSITY EXAMINATIONS 2016/2017

**FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE
CERTIFICATE IN BRIDGING MATHEMATICS**

CITY CAMPUS

MMA 002: VECTORS AND GEOMETRY

Date: 22nd November 2016

Time: 2.00 - 5.00pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.



QUESTION ONE (Compulsory)

[30 Marks]

- (a) If $A = \{a, b, c\}$ and $B = \{d, e\}$, find:
- (i) $\mathcal{P}(A)$, the power set of A . [2 Marks]
 - (ii) $A \times B$. [2 Marks]
- (b) Evaluate the following:
- (i) $(-2, 3, 5) - 3(-7, -4, 8)$ [2 Marks]
 - (ii) $-2(5, -8, 4) + 4(-4, 0, -2)$ [2 Marks]
- (c) (i) Differentiate between additive identity and additive inverse of a vector. [2 Marks]
- (ii) Find the additive identity and additive inverse of the vector $\underline{i} + 3\underline{j} - 4\underline{k}$. [2 Marks]
- (d) (i) Differentiate between linear independence and linear dependence of vectors. [2 Marks]
- (ii) Consider the vectors $\underline{u} = (1, 2, -1)$ and $\underline{v} = (6, 4, 2)$ in \mathbb{R}^3 . Show that $\underline{w} = (9, 2, 7)$ is a linear combination of \underline{u} and \underline{v} . [3 Marks]
- (e) Given that vectors $\underline{u} = 3\underline{i} - 4\underline{j} - 2\underline{k}$ and $\underline{v} = \underline{i} - 4\underline{j} + \underline{k}$, find:
- (i) $\underline{u} \cdot \underline{v}$ [3 Marks]
 - (ii) $\underline{u} \times \underline{v}$ [4 Marks]
- (f) Normalize the following vectors:
- (i) $\underline{i} - 3\underline{j} + \underline{k}$ [3 Marks]
 - (ii) $3\underline{i} + 4\underline{j} - 2\underline{k}$ [3 Marks]

QUESTION TWO

[20 Marks]

- (a) Consider the vectors $\underline{u} = (1, 2, -1)$ and $\underline{v} = (6, 4, 2)$ in \mathbb{R}^3 . Show that $\underline{w} = (0, 8, -8)$ is a linear combination of \underline{u} and \underline{v} , and that $\underline{w}' = (4, -1, 8)$ is not a linear combination of \underline{u} and \underline{v} . [6 Marks]
- (b) Use dot product to find the sides of the parallelogram with co-ordinates $(1, 2)$, $(7, 2)$, $(4, 4)$ and $(10, 4)$. [8 Marks]
- (c) Determine whether the vectors $\underline{i} = (1, 0, 0)$, $\underline{j} = (0, 1, 0)$ and $\underline{k} = (0, 0, 1)$ are linearly dependent or linearly independent. [6 Marks]

QUESTION THREE

[20 Marks]

Let $u = (3, 2, -1)$, $v = (0, 2, -3)$ and $w = (2, 6, 7)$. Compute

- (a) $v \cdot (4u - w)$ [4 Marks]
(b) $u \times (v - 2w)$ [6 Marks]
(c) $(u \times v) \times (v \times w)$ [10 Marks]

QUESTION FOUR

[20 Marks]

(a) A man wishes to empty a cylindrical water tank with a diameter of 10 m. The outlet valve is on the opposite side of the tank, due north of him. Obtain:

- (i) the displacement of the valve from his present location, and
(ii) the minimum distance he must walk in order to open the valve.

[3 Marks]

(b) Let $u = (2, -2, 3)$, $v = (1, -3, 4)$ and $w = (3, 6, -4)$. Evaluate the expression:

- (i) $|u + v + w|$ [3 Marks]
(i) $|-2u| + 2|u|$ [3 Marks]
(ii) $|3u - 5v + w|$ [3 Marks]
(iii) $\left| \frac{1}{|w|} w \right|$ [3 Marks]

(c) Use diagrams to show the following:

- (i) $u + v = v + u$ [2 Marks]
(ii) $u + (v + w) = (u + v) + w$ [3 Marks]

QUESTION FIVE

[20 Marks]

(a) Differentiate between:

- (i) commutative and associative property of vectors. [2 Marks]
(ii) position vector and displacement vector. [2 Marks]

(b) Find the distance between P_1 and P_2 .

- (i) $P_1(-3, 6)$ and $P_2 = (-1, -4)$. [2 Marks]
(ii) $P_1(7, -5, 1)$ and $P_2 = (-7, -2, -1)$. [3 Marks]

(c) Let $y = (-1, 2, 5)$. Find all scalars k such that $|ky| = 4$. [6 Marks]

(d) For which values of k do the following vectors become linearly dependent.

$$u = \left(k, -\frac{1}{2}, -\frac{1}{2}\right), \quad v = \left(-\frac{1}{2}, k, -\frac{1}{2}\right), \quad w = \left(-\frac{1}{2}, -\frac{1}{2}, k\right) \quad [5 \text{ Marks}]$$