MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE (ECOHIM) WITH INFORMATION TECHNOLOGY

CITY CAMPUS

MMA 107: MATHEMATICS II

Date: 30th November, 2016 Time: 9.00 - 12.00pm

INSTRUCTIONS:

- Answer Questions ONE (Compulsory) and any other TWO.
- Observe further instructions on the answer booklet

Question 1 (30 Marks)

a) Show that $(4+3\sqrt{x})^2$ can be written as $16+k\sqrt{x}+9x$, where k is a constant to be found (3 Marks)

b) Find $\int(4+3\sqrt{x})^2 dx$ (6 Marks)

c) The function f is defined by

$$f: x \to \frac{2x+3}{x-1} \{x \in \mathbb{R}, x > 1\}$$

Find

i) $f^{-1}(x)$ (6 Marks)

ii) the range of $f^{-1}(x)$ (3 Marks)

iii) the domain of $f^{-1}(x)$ (2 Marks)

d) Given $f(x) = x^3 + 3x^2 + 5$ find

i) $f''(x)$ (4 Marks)

ii) $\int_1^2 f(x)dx$ (6 Marks)

Question 2 (20 Marks)

The curve C has equation

$$y = 6 - 3x - \frac{4}{x^3}, x \neq 0$$

a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$ (5 Marks)

b) Find the x-coordinate of the other turning point Q on the curve. (3 Marks)

c) Find

$$\frac{d^2y}{dx^2}$$

(2 Marks)

Hence or otherwise, state with justification, the nature of each of these turning points P and Q. (4 Marks)

d) On the graph paper provided, sketch the curve C (6 Marks)

Question 3 (20 Marks)

A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi\ cm^3$. The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas. Given that the radius of the cylinder is r cm,

a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(10 Marks)

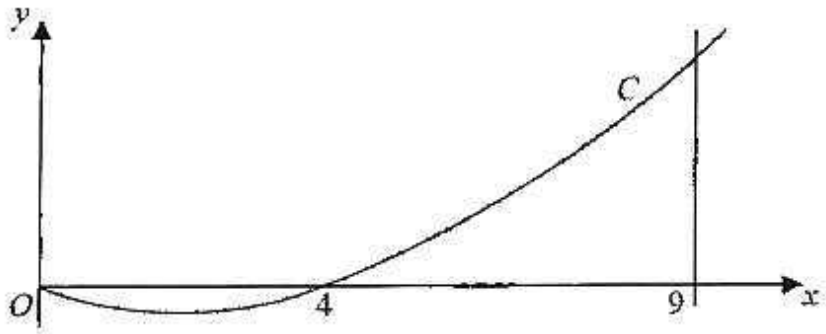
b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. (8 Marks)

c) Justify that the answer that you have obtained in part (b) is a minimum. (2 Marks)

Question 4 (20 Marks)

a) Given $f(x) = sin(2x^3)$ determine $f'(x)$ (6 Marks)

b) Find $\int 10x\left(x^{\frac{1}{2}} - 2\right)dx$ giving each term in its simplest form (6 Marks)



c) The Figure above shows a sketch of part of the curve C with equation

$$y = 10x\left(x^{\frac{1}{2}} - 2\right), x \geq 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0). The area, shown shaded in Figure, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$. Use your answer from part (a) to find the total area of the shaded regions. (8 Marks)

Question 5 (20 Marks)

a) Given $f(x) = 3xe^{x^3}$, determine $f'(x)$ (5 Marks)

b) On the graph paper provided, sketch the curve $y = x^3 - 6x^2 + 5x$ (7 Marks)

c) Use calculus to find the total area under the curve bounded by $x = 0$ and $x = 2$ (8 Marks)