MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 111: INTRODUCTION TO CALCULUS

Date: 29th November, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.

Question One [30marks]

a) Find the domain of the function $f(x) = \sqrt{2-x} + \sqrt{x+5}$ [4mks]

b) Evaluate the following limits

i) $\lim_{x\to 0} \frac{\tan 4x}{\sin 5x}$ [4mks]

ii) $\lim_{x\to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5}$ [3mks]

c) Let $f(x) = \sqrt{4-3x}$. Use the definition of the derivative to find $f'(x)$ [5mks]

d) Determine whether the function

$$f(x) = \begin{cases} x^2 - x + 2, & \text{if } x \leq 2 \\ \frac{1}{x+2}, & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$. [3mks]

e) A retailer has determined that the cost C for ordering and storing x units of a product is

$$C = 2x + \frac{300,000}{x}, \quad 1 \leq x \leq 300$$

The delivery truck can bring at most 300 units per order. Find the order size that will minimize cost. [4mks]

f) Find the derivative of the following functions.

i) $f(x) = \frac{x+1}{x-1}$ [3mks]

ii) $f(x) = x\cos^2(1-x)$ [4mks]

Question Two [20 marks]

a) Use the sandwich Theorem to determine the value of [6mks]

$$\lim_{x\to 0} x^4 \sin\left(\frac{\pi}{x}\right)$$

b) Use the intermediate value Theorem to prove that the equation $3x^5 - x^3 = 1$ has a solution in the interval $(0, 1)$. [6mks]

c) If $g(x) \neq 0$, show that the derivative of the quotient $k(x) = \frac{f(x)}{g(x)}$ of two differentiable functions f and g is given by [8mks]

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Question Three [20 marks]

a) If $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ where a and b are constants, find $\frac{dy}{dx}$ [7mks]

b) Find the tangent line to the curve $x^2 + xy - y^2 = 1$ at the point $(2,3)$ [7mks]

c) Find $\frac{dy}{dx}$ if [6mks]

$y = \sin^{-1}(\sqrt{\cos x})$

Question Four [20 marks]

a) Find derivatives of the following functions

i) $f(x) = x^{x+2}$ [3mks]

ii) $g(x) = \ln(x^2 + 6)$ [2mks]

iii) $h(x) = e^{\frac{-x^2}{2}}$ [3mks]

iv) $y = \sqrt[3]{(x^2-1)^2}$ [3mks]

b) A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are each $1\frac{1}{2}$ inches. The margins on each side are 1 inch. What should the dimensions of the page be so that the least amount of paper is used? [9mks]

Question Five [20marks]

a) i) Use L'Hospital's Rule to evaluate [3mks]

$$\lim_{x \to -\infty} \frac{x^2}{e^{1-x}}$$

ii) Two stationary patrol cars equipped with radar are 5 miles apart on a highway. As a car passes the first patrol car, its speed is clocked at 55 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Prove that the truck must have exceeded the speed limit of 55 miles per hour at some time during the 4 minutes. [5mks]

b) Let $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$

i) Find the intervals on which f is increasing or decreasing. [4mks]

ii) Find the local maximum and minimum values of f. [2mks]

iii) Find the intervals of concavity and the inflection point. [3mks]

iv) Sketch the graph of f. [3mks]

END