MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 203: CLASSICAL MECHANICS

Date: 30th November, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Show all the necessary workings
- Start each question on a new page.
- Observe further instructions on the answer booklet.

QUESTION ONE (COMPULSORY) (30mks)

(a) (i) Distinguish between a conservative and a non-conservative force field [2mks]

(ii) Given $\mathbf{F} = (2xy + z^3)i + x^2j + 3xz^2k$

(I) Show that $\mathbf{F}$ is conservative. [3mks]

(II) Find the potential energy. [4mks]

(b) (i) A particle is dropped from rest at height h. With what velocity does it strike the ground? [2mks]

(ii) Find the work done by the force field $\mathbf{F}(x, y) = (x \sin y, y)$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$ [4 mks]

(c) The trajectory of a charged particle in a magnetic field is given by

$$\underline{r} = b\cos\omega t i + b\sin\omega t j + ctk$$

where b, ω and c are positive constants. Show that the particle moves with constant speed and find the magnitude of its acceleration. [6mks]

(d) A particle of mass m moves along the ellipse $\vec{r} = a\cos wt\vec{i} + b\sin wt\vec{j}$ under the influence of a force field $\mathbf{F}$, prove that $\vec{r} \times \mathbf{F}$ [4mks]

(e) An object of mass m is dropped from a height H above the ground. Prove that if the air resistance is negligible, then it will reach the ground in time $\sqrt{\frac{2H}{g}}$ [5mks]

QUESTION TWO (20mks)

(a) If r is the position vector of a particle of mass m relative to point O and F is the external force on the particle, then $\mathbf{r} \times \mathbf{F} = \mathbf{M}$ is the torque or moment of F about O. Show that $\mathbf{M} = \frac{dH}{dt}$, where $H = \mathbf{r} \times m\mathbf{v}$ and v is the velocity of the particle. [4mks]

(b) A mass of 5000Kg moves on a straight line from a speed of 540Km/hr to 720Km/hr in 2 minutes. What is the impulse developed at this time? [3mks]

(c) A particle of mass m moves under the influence of the force given by

$$\vec{F} = a(\sin \omega t i + \cos \omega t j)$$

If the particle is initially at rest at the origin, prove that the work done on the particle upto time (t) is given by

$$W = \frac{a^2}{mw^2}(1 - \cos \omega t)$$

[6mks]

(d) A particle P moves along the $x-$ axis with constant acceleration a in the positive $x-$ direction. Initially P is at the origin and is moving with velocity u in the positive. Show that the velocity v and displacement $\vec{x}$ of P at time t are given by

$$\underline{v} = u + at, \quad \underline{x} = ut + \frac{1}{2}at^2$$

and deduce that $\underline{v^2} = u^2 + 2ax$. [7mks]

QUESTION THREE (20mks)

(a) Find the work done in moving a particle in the force field:

$\mathbf{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$

(i) along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$ [5 mks]

(ii) along the space curve $x = 2t^2; y = t; z = 4t^2 - t$ from $t = 0$ to $t = 1$. From the results in (i) and (ii), what can you conclude. [5mks]

(b) A particle moves along a straight line and its distance from a fixed point on the line is given by $\vec{X} = a\cos(\omega t + \varepsilon)$. Show that its acceleration varies as the distance from the origin and is directed towards the origin [4mks]

(c) A particle P of mass 4 kg moves under the action of the force $\mathbf{F} = 4i + 12t^2j$**Newtons**, where t is the time in seconds. The initial velocity of the particle is $2i + j + 2k\ ms^{-1}$. Find the work done by $\mathbf{F}$ and the increase in Kinetic Energy of P, during the time interval $0 \leq t \leq 1$. What principle does this illustrate? [6mks]

QUESTION FOUR (20mks)

(a) An object of mass 20kg moves with SHM on the x- axis. Initially, its located at a distance 4m away from origin with a velocity $15m/s$ and acceleration of $100m/s^2$ directed towards the origin. Find;

(i) the position at any time

(ii) the amplitude, period and frequency [8mks]

(b) A particle of mass 5 units moving along the X axis is attracted towards origin by a force whose magnitude is numerically equal to $40x$. It is initially at rest at $x = 20$, and has also a damping force proportional to the instantaneous speed such that when the speed is 10 units/sec, a damping force is 200 units.

(i) Set up a differential equation and initial conditions describing the motion [3mks]

(ii) Find the position of the particle at any time t. [7mks]

(iii) Determine the amplitude, period and frequency of the damped oscillator. [2mks]

QUESTION FIVE (20mks)

. projectile is launched at an angle α from a cliff of height H above sea level. it falls into the sea at a distance D from the base of the cliff, prove that s maximum height above sea level is

$$H + \frac{D^2 \tan^2 \alpha}{4(H + D \tan \alpha)}$$

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