# MASENO UNIVERSITY

## UNIVERSITY EXAMINATIONS 2016/2017

## SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

## MAIN CAMPUS

## MMA 210: PROBABILITY AND DISTRIBUTION THEORY I

Date: 12th December, 2016 Time: 3.30 - 6.30 pm

**INSTRUCTIONS:**

- Answer question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.
- Observe further instructions on the answer booklet.

**MMA210: PROBABILITY AND DISTRIBUTION THEORY I**

**INSTRUCTION:** Answer Question ONE and TWO other Questions

**QUESTION ONE (30MKS)**

a). If $X \sim x^2$ (8), determine the constants $a$ and $b$ so that

i. $P(a < X < b) = 0.95$ and [3mks]

ii. $P(X < a) = 0.025$. [2mks]

b). Suppose that $X$ and $Y$ have a continuous joint density given by

$$f(x, y) = 12x^2 \quad 0 \leq x \leq y \leq 1$$

Find the

i. Mean [2mks]

ii. Variance [3mks]

c). The annual proportion of erroneous income tax returns filed by the income tax department is thought to be a random variable having a beta distribution with parameters $\alpha = 2$ and $\beta = 9$. What is the probability that in any given year, there will be fewer than 10% erroneous returns? [4mks]

d). Suppose that $X \text{ and } Y$ have a discrete joint probability function given by

$$f(x, y) = \begin{cases} \frac{1}{30}(x + y) & x = 0,1,2 \\ & y = 0,1,2,3 \\ 0 \text{ elsewhere} & \end{cases}$$

Determine the marginal probability functions of $X \text{ and } Y$ [4mks]

e). Let $X \text{ and } Y$ have the bivariate density

$$f(x, y) = e^{-x-y} \quad x > 0, y > 0$$

Determine the conditional density of $X$ given $Y = y$ [4mks]

f). Let $X$ and $Y$ have the joint p.d.f

$$f(x, y) = \begin{cases} x + y & 0 < x < 1, \quad 0 < y < 1 \\ 0 \text{ elsewhere} \end{cases}$$

Determine whether or not $X$ and $Y$ are independent. [4mks]

f). Let $X$ and $Y$ have the joint p.d.f

$$f(x,y) = \begin{cases} x+y & 0<x<1, \quad 0<y<1 \\ 0 & elsewhere \end{cases}$$

Determine whether or not $X$ and $Y$ are independent. [4mks]

g). Suppose that $f(x,y) = x+y \quad 0<x<1,\ 0<y<1$

Compute $E(XY)$ [4mks]

## QUESTION TWO (20MKS)

a). Suppose that a continuous random variable $X$ has the chi-squared distribution with $n$ degrees of freedom given by

Show that [9mks]

i. $E(X) = n$

ii. $Var(X) = 2n$

iii. The m.g.f of is $X$is $m_x(t) = \frac{1}{(1-2t)^{\frac{n}{2}}}$

b). Determine the mode of the gamma distribution with parameters $\alpha$ and $\beta$; [3mks]

c). The family income in a certain urban area (in thousands of shillings) is believed to be a random variable having a gamma distribution with parameters $\alpha = 2$ and $\beta = 4$.Compute the probabilities that a family, randomly selected from this area , will have an income less than $Kshs$ 4000 [4mks]

d). Suppose that $X$ is continuous random variable with p.d.f

$$f(x) = 3x^2 \ ,0<x<1$$

Find the p.d.f of $Y = 8x^3$ [4mks]

## QUESTION THREE (20MKS)

a). A carton contains 18 vials of different brands of perfume. Of these 6

**QUESTION THREE (20MKS)**

a). A carton contains 18 vials of different brands of perfume. Of these 6 are "*channel* 5" ,5 are "*brut*" and 7 are "*Roche*". Let $X$ denote the number of" channel 5" and $Y$ denote the number of "Roche" vials, in a random sample of 4 vials selected from the carton. Determine the joint probability of $X$ and $Y$. [6mks]

b). Suppose that $X$ *and* $Y$ are continuous random variables such that $(X,Y)$ takes the values only in the rectangle $\{(X,Y): 0 \le x \le 2, 0 \le y \le 1\}$. Suppose also that the joint distribution function of $X$ *and* $Y$ at any point $(x,y)$ in the rectangle is given by

$$F(X,Y) = \frac{1}{10}xy(x^2+y^2)$$

Determine

i. $P\left(\frac{1}{3} \le X \le 1, \frac{1}{4} \le Y \le \frac{1}{2}\right)$ [2mks]

ii. $P\left(1 \le X \le 3, \frac{1}{2} \le Y \le 1\right)$ [2mks]

iii. The c.d.f of $X$ [2mks]

iv. The joint p.d.f of $X$ *and* $Y$ and $P(Y \ge X)$ [4mks]

c). Suppose that the joint p.d.f of $X$ and $Y$ is [4mks]

$f(x,y) = 2e^{-(x+2y)} \quad x > 0, y > 0$

Show that $X$ and $Y$ are independent

**QUESTION FOUR (20MKS)**

a). Given the joint probability density

$$f(x,y) = \frac{2}{(1+x+y)^3} \quad x > 0, y > 0$$

Find

i. The marginal density of $X$ and $Y$ [4mks]

ii. The conditional density of $X$ and $Y = y$ [2mks]

iii. Are $X$ and $Y$ independent [2mks]

b). Let $X$ and $Y$ are discrete random variables with joint probability function

$$f(x,y) = 21x^2y^3 \quad 0 \le x \le y \le 1$$

Determine the conditional mean and variance of $X$ given $Y = y, 0 < y < 1$ [6mks]

c). Let $X$ and $Y$ have the joint distribution specified by $f(x,y) = \frac{xy}{36}$ $x = 1,2,3$ $y = 1,2,3$

Find the distribution of $U = XY$ and $= Y$ . Hence or otherwise find the marginal distribution $U$ [6mks]

**QUESTION FIVE (20MKS)**

a). Consider an experiment of tossing two regular tetrahedral. Let $X$ be the number on the first and $Y$ the larger of the two numbers. [10mks]

Compute $E(X)$, $E(Y)$ ,$E(XY)$, $E(2X+Y)$

b). If $X$ and $Y$ are two discrete random variables having joint distribution

| $(X,Y)$ | (0,0) | (0,1) | (1,0) | (1,1) | (2,0) | (2,1) |
|---|---|---|---|---|---|---|
| | $\frac{1}{18}$ | $\frac{3}{18}$ | $\frac{4}{18}$ | $\frac{3}{18}$ | $\frac{6}{18}$ | $\frac{1}{18}$ |

Find the conditional means [4mks]

c). Suppose that $X$ is continuous random variable with p.d.f

$f(x) = \frac{2}{25}(x+2), -2 < x < 3$

Find the p.d.f of $Y = X^2$ [3mks]

d). Let $X$ have the probability distribution $f(x) = \frac{1}{3}$ $x = 1,2,3$

Find the distribution of $Y = 2X + 1$ [4mks]