MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 215: CALCULUS

Date: 12th December, 2016   Time: 12.00 - 3.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Show all the necessary workings
- Start each question on a new page.
- Observe further instructions on the answer booklet.

Question One [30marks]

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a) i) Use part one of the Fundamental Theorem of Calculus to find $\frac{dy}{dx}$ if [3mks]

$$y = \int_1^{x^2} \cos t \, dt$$

ii) Use the definition of the definite integral to evaluate the integral [5mks]

$$\int_1^4 (2x + 3) dx$$

b) Evaluate the following integrals

i) $\int \frac{4x+3}{4x^2+6x-1} dx$ [3mks]

ii) $\int t^2 e^t dt$ [4mks]

c) Find the arc length of the graph of $(y-1)^3 = x^2$ on the interval $[0, 8]$. [5mks]

d) Evaluate the improper integral [5mks]

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$$

e) Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$. [5mks]

Question Two [20 marks]

a) Use integration by parts to show that [8mks]

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx,$$

hence evaluate $\int \sin^3 x \, dx$

b) Evaluate the following integrals

i) $\int \frac{x^3}{\sqrt{9-x^2}} dx$ [5mks]

ii) $\int \frac{x^3-x^2-7x+8}{x^2-4x+4}$ [7mks]

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Question Three [20 marks]

a) Approximate the value of π using the Simpson's approximations of

$$\int_0^1 \frac{4}{1+x^2}\,dx$$

for $n = 4$, Estimate also the error. [10mks]

b) Evaluate the following definite integrals

i) $\int_0^4 f(t)\,dt$ where [4mks]

$$f(t) = \begin{cases} 2t, & \text{if } t > 1 \\ 1 - 3t^2, & \text{if } t \leq 1 \end{cases}$$

ii) $\int_3^6 |2x - 10|\,dx$ [6mks]

Question Four [20 marks]

a) An electric cable is hung between two towers that are 200 feet apart. The cable takes the shape of a catenary whose equation is

$$y = 75 \left(e^{\frac{x}{150}} + e^{\frac{x}{150}}\right), -100 \leq x \leq 100.$$

Find the arc length of the cable between the two towers. [6mks]

b) For what values of p is the following integral convergent? [8mks]

$$\int_1^\infty \frac{1}{x^p}\,dx.$$

c) Find the volume of the solid obtained by rotating the region bounded by $y = x^3 + x + 1$, $x = 1$ and $y = 1$ about the line $x = 2$. [6mks]

Question Five [20marks]

a) Phenomena such as waiting times and equipment failure times are commonly modelled by exponentially decreasing probability density functions of the form

$$f(t) = \begin{cases} 0, & \text{if } t < 0 \\ ce^{-ct}, & \text{if } t \geq 0 \end{cases}$$

i) Find the mean μ of the exponential distribution. (Take $\mu = \int_{-\infty}^\infty t f(t)\,dt$) [5mks]

ii) Suppose the average waiting time for a customer's call to be answered by a company representative is five minutes($\mu = 5$), find the probability that a call is answered during the first minute.

[4mks]

b) Assume that in a certain city the temperature (in $^\circ$F) t hours after 9 A.M is represented by the function

$$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$$

Find the average temperature in that city during the period 9 A.M to 9 P.M.

[4mks]

c) Evaluate the following indefinite integrals

i) $\int \ln x\, dx$

[3mks]

ii) $\int \sin^5 x \cos^2 x\, dx$

[4mks]

END

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