MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 307: METHODS I

Date: 7th December, 2016 Time: 8.30 - 11.30am

INSTRUCTIONS:

- Answer Question ONE and any other TWO.

QUESTION ONE (Compulsory) [30 Marks]

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(a) Prove that

$$\int xJ_0^2(x)\mathrm{d}x = \frac{1}{2}x^2[J_0^2 + J_1^2] + c.$$

[5 Marks]

(b) Express $f(x) = x^3$ in terms of Legendre polynomials. [5 Marks]

(c) Using the result that

$$\int_0^\infty e^{-ax}x^{n-1}\cos bx\mathrm{d}x = \frac{\Gamma(n)\cos n\theta}{(a^2+b^2)^{\frac{n}{2}}}, \quad \theta = \tan^{-1}\frac{b}{a},$$

evaluate:

(i) $\int_0^\infty \cos x^2\mathrm{d}x$. [9 Marks]

(ii) $\int_0^\infty \cos\frac{\pi x^2}{2}\mathrm{d}x$. [5 Marks]

(d) Find the Laplace transform of:

(i) $\dfrac{1-e^t}{t}$, [3 Marks]

(ii) $\dfrac{\cos 2t - \sin 3t}{t}$. [3 Marks]

QUESTION TWO [20 Marks]

Assuming a solution of the form:

$$y = \sum_{k=0}^{\infty} a_k(x-2)^k$$

and rewriting

$$1 - x = -[(x-2)+1]$$
$$2x = 2(x-2)+4,$$

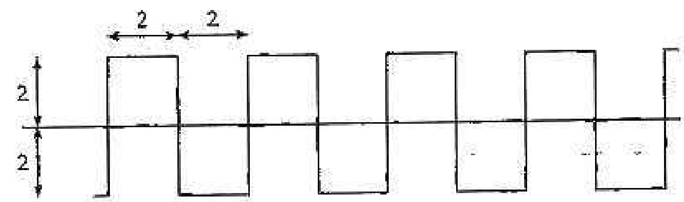
solve giving the first four terms the equation:

$$(1-x)\frac{\mathrm{d}f(x)}{\mathrm{d}x} - f(x) = 2x$$

such that $f(2) = 1$.

QUESTION THREE [20 Marks]

A periodic forcing function acts on a spring-mass system as shown:



Find:

(a) A sine-series representation by considering the function to be odd. [10 Marks]

(b) A cosine-series representation by considering the function to be even. [10 Marks]

QUESTION FOUR [20 Marks]

Solve

$$t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \cos t$$

if $y(0) = 0, y'(0) = 0$ using the Laplace transform method. [20 Marks]

QUESTION FIVE [20 Marks]

Solve the following equation by the method of separation of variables:

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$$

[20 Marks]