JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL EXAMINATION

4th YEAR 2nd SEMESTER 2017/2018ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA414

COURSE TITLE: FOURIER ANALYSIS

EXAM VENUE: **STREAM: (BSc/BED)**

DATE: 24/05/18 ***EXAM SESSION: 2.00 – 4.00PM***

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question.1 [30 marks] Compulsory

(a) Assume that $f(x)$ has a uniformly convergent Fourier series

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}, \quad -\pi \le x \le \pi .$$

Prove that

(i) $\int_{-\pi}^{\pi} f(x)dx = \dfrac{\pi a_0}{2}$

(ii) $a_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$

(iii) $b_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx$

iv) $\dfrac{1}{\pi}\int_{-\pi}^{\pi} (f(x))^2 dx = \dfrac{a^2{}_0}{2} + \sum_{n=1}^{\infty} \{a^2{}_n + b^2{}_n\}$ [11 marks]

(b) Given the function $f(x) = 4x \quad , -50 < x < 50 \quad f(x) = f(x+100)$

(i) Sketch graph of $f(x)$ over the interval $-400 < x < 400$

(ii) State period of $f(x)$ [9 marks]

(c) For the step function

$$f(x) = \begin{cases} 2\pi & ; \quad 0 \le x < \pi \\ -2\pi & ; \quad -\pi \le x < 0 \end{cases}$$

deduce that $f(x)$ may be expressed in an infinite series

$$f(x) = \frac{1}{\pi}\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{n}\sin nx$$ [10marks]

Question2 [20 marks]

(a) Assume that $f(x)$ has a uniformly convergent Fourier series

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}, \quad -\pi \le x \le \pi .$$

Discuss the Gibb's Phenomenon of $f(x)$. [8 marks]

b) One cycle of a periodic waveform $y = f(x)$ of period 2π is defined by the below data.

x^0	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$	15	20	23	24	20	8	3	4	9	12	10	11

Determine the approximate Fourier series for $y = f(x)$ up to and including the third harmonic. [12 marks]

Question3 [20 marks]

Given real valued function $y = f(x)$ for which

$$f(x)=\begin{cases}0 & -\pi < x < 0 \\ -x & 0 < x < \pi\end{cases}$$

$$f(x)=f(x+2\pi)$$

(a)Sketch the graph of $f(x)$ over the interval $-4\pi < x < 4\pi$ [5 marks]

(b)State period of $f(x)$ [3 marks]

(c)Show that

$$f(x)=\frac{\pi}{4}+\frac{2}{\pi}\left\{\cos x+\frac{1}{9}\cos 3x+\frac{1}{25}\cos 5x+\frac{1}{49}\cos 7x\ldots\right\} +\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-\frac{1}{4}\sin 4x\ldots$$

[12 marks]

Question4 [20 marks]

The centre of an elastic string 4m long is deflected initially to a point P 0.01m high and then released. The initial velocity is zero. Given u the wave displacement of the string satisfies the equation

$u_{xx}=\frac{1}{100}u_{tt}$, $u=u(x,t)$ Subject to boundary and initial conditions

$$u(x,0)=\begin{cases}\dfrac{x}{200} & 0 < x < 2 \\ \dfrac{2}{100}-\dfrac{x}{200} & 2\ 0 < x < 4\end{cases}$$

$u_t(x,0)=0\ ;0 < x < 4\ ,u(0,t)=u(0,4)=0\ ;0 < t$, determine the wave movement of the string

[20 marks]

Question5 [20 marks]

(a) Consider the function defined on $[0,\ \pi)$ by

$$f(x)=\begin{cases}2x & 0 \le x < \dfrac{\pi}{2} \\ \pi-2x & \dfrac{\pi}{2} \le x < \pi\end{cases}$$

(i) Obtain the Fourier **half-range** cosine coefficients for this function $f(x)$.

(ii)Sketch the symmetric even periodic extension of f(x) on $(-\pi, \pi)$

$f(x)=10$, $0 < x < \pi$ [10 marks]

(b) Let $f(x)$ be periodic function with period 2π with

$f(x)=|\sin 2x|$, $-\pi \le x \le \pi$

(i) State giving reasons whether $f(x)$ is odd or even.

(ii) Sketch $f(x)$ over the interval $[-4\pi,\ 4\pi]$

(iii)Obtain the Fourier series expansion for $f(x)$ [10 marks]