MASENO UNIVERSITY
UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 211: PROBABILITY AND DISTRIBUTION THEORY

Date: 7th July, 2018 Time: 8.30 – 11.30 am

INSTRUCTIONS

- This paper consist of TWO sections (section I and Section II)
- Answer ALL the questions in SECTION I and any other TWO questions in SECTION II
- In each question show your clear working on how your arrive at the answers
- There will be mark for proper workings even if the final answer is wrong

SECTION I: Answer ALL the questions in this section

QUESTION ONE - 30 MARKS (COMPULSORY)

(a) State four reasons for sampling [4 mks]

(b) A particular brand of drink has an average of 12 ounces per can. As a result of randomness, there will be small variations in how much liquid each bottle really contains. It has been observed that the amount of liquid in these bottles is normally distributed with $\sigma = 0.8$ ounce. A sample of 10 bottles of this brand of soda is randomly selected from a large lot of bottles, and the amount of liquid, in ounces, is measured in each. Find the probability that the sample mean will be within 0.5 ounce of 12 ounces. [5 mks]

(c) A company that manufactures cars claims that the gas mileage for its new line of hybrid cars, on the average, is 60 miles per gallon with a standard deviation of 4 miles per gallon. A random sample of 16 cars yielded a mean of 57 miles per gallon. If the companys claim is correct, what is the probability that the sample mean is less than or equal to 57 miles per gallon? Comment on the companys claim about the mean gas mileage per gallon of its cars. [3 mks]

(d) If a random variable X has a gamma distribution with parameters α and β, then prove that $Y = \frac{2X}{\beta} \sim \chi^2(2\alpha)$. [Hint: mgf of gamma random variable X is $(1-\beta t)^{-\alpha}$] [4 mks]

(e) Let the random variables $X_1, X_2, ..., X_5$ be from an $N(5,1)$ distribution. Find a number a such that

$$P\left(\sum_{1=i}^{5}(X_i - 5)^2 \leq a\right) = 0.90$$

[5 mks]

QUESTION TWO - 20 MARKS

(a) Let a population consist of the numbers 1, 2, 3, 4, 5. Consider all possible samples consisting of three numbers randomly chosen without replacement from this population.

(i) Obtain the distribution of the sample mean.

(ii) Calculate the mean and Variance of $\bar{X}$

(b) Let X has a Chi-square distribution with degrees of freedom n if its probability density function is

$$f(x) = \begin{cases} \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} exp(-\frac{1}{2}x) & x > 0 \\ 0 & Otherwisw \end{cases}$$

find $E(X)$

QUESTION THREE - 20 MARKS

Let $X_1, \ldots, X_n$ be a random sample of size n from a population with mean μ and variance σ^2.

(i) Show that $E(X) = \mu$ and $Var(X) = \frac{\sigma^2}{n}$.

(ii) Consider the sample variance

$$S^2 = \frac{1}{n-1}\sum_{1=i}^{n}(X_i - \bar{X})^2$$

Show that $E(S^2) = \sigma^2$

QUESTION FOUR - 20 MARKS

(a) Let $X_1, ..., X_k$ be independent χ^2 random variables with $n_1, ..., n_k$ degrees of freedom, respectively. Use mgf to show that the sum $V = \sum_{1=i}^{n} X_i$ is chi-square distributed with $n_1 + n_2 + \ + n_k$ degrees of freedom.

(b) Suppose that $X \sim \chi^2$ random variable with 20 degrees of freedom. Use the chi-square table to obtain the following:

(i) Find x_0 such that $P(X > x_0) = 0.95$.

(ii) Find P $(X \leq 12.443)$.

(c) Let $X_1, ..., X_k$ be a random sample from a normal distribution with $\sigma^2 = 0.8$. Find two positive numbers a and b such that the sample variance S^2 satisfies $P(a \leq S^2 \leq b) = 0.90$

QUESTION FIVE - 20 MARKS

(a) If $\bar{X}$ and S_2 are the mean and the variance of a random sample of size n from a normal population with the meanμ and variance σ^2, show that $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ has a t-distribution with $(n-1)$ degree of freedom

(b) Let S_1^2 denote the sample variance for a random sample of size 10 from Population I and let S_2^2 denote the sample variance for a random sample of size 8 from Population II. The variance of Population I is assumed to be three times the variance of Population II. Find two numbers a and b such that $P(a \leq S_1^2/S_2^2 \leq b) = 0.90$ assuming S_1^2 to be independent of S_2^2

Table of the chi square distribution – Appendix J, p. 915

	Level of Significance α								
df	0.200	0.100	0.075	0.050	0.025	0.010	0.005	0.001	0.0005
1	1.642	2.706	3.170	3.841	5.024	6.635	7.879	10.828	12.116
2	3.219	4.605	5.181	5.991	7.378	9.210	10.597	13.816	15.202
3	4.642	6.251	6.905	7.815	9.348	11.345	12.838	16.266	17.731
4	5.989	7.779	8.496	9.488	11.143	13.277	14.860	18.467	19.998
5	7.289	9.236	10.008	11.070	12.833	15.086	16.750	20.516	22.106
6	8.558	10.645	11.466	12.592	14.449	16.812	18.548	22.458	24.104
7	9.803	12.017	12.883	14.067	16.013	18.475	20.278	24.322	26.019
8	11.030	13.362	14.270	15.507	17.535	20.090	21.955	26.125	27.869
9	12.242	14.684	15.631	16.919	19.023	21.666	23.589	27.878	29.667
10	13.442	15.987	16.971	18.307	20.483	23.209	25.188	29.589	31.421
11	14.631	17.275	18.294	19.675	21.920	24.725	26.757	31.265	33.138
12	15.812	18.549	19.602	21.026	23.337	26.217	28.300	32.910	34.822
13	16.985	19.812	20.897	22.362	24.736	27.688	29.820	34.529	36.479
14	18.151	21.064	22.180	23.685	26.119	29.141	31.319	36.124	38.111
15	19.311	22.307	23.452	24.996	27.488	30.578	32.801	37.698	39.720
16	20.465	23.542	24.716	26.296	28.845	32.000	34.267	39.253	41.309
17	21.615	24.769	25.970	27.587	30.191	33.409	35.719	40.791	42.881
18	22.760	25.989	27.218	28.869	31.526	34.805	37.157	42.314	44.435
19	23.900	27.204	28.458	30.144	32.852	36.191	38.582	43.821	45.974
20	25.038	28.412	29.692	31.410	34.170	37.566	39.997	45.315	47.501
21	26.171	29.615	30.920	32.671	35.479	38.932	41.401	46.798	49.013
22	27.301	30.813	32.142	33.924	36.781	40.289	42.796	48.269	50.512
23	28.429	32.007	33.360	35.172	38.076	41.639	44.182	49.729	52.002

24	29.553	33.196	34.572	36.415	39.364	42.980	45.559	51.180	53.480
25	30.675	34.382	35.780	37.653	40.646	44.314	46.928	52.620	54.950
26	31.795	35.563	36.984	38.885	41.923	45.642	48.290	54.053	56.409
27	32.912	36.741	38.184	40.113	43.195	46.963	49.645	55.477	57.860
28	34.027	37.916	39.380	41.337	44.461	48.278	50.994	56.894	59.302
29	35.139	39.087	40.573	42.557	45.722	49.588	52.336	58.302	60.738
30	36.250	40.256	41.762	43.773	46.979	50.892	53.672	59.704	62.164
40	47.269	51.805	53.501	55.759	59.342	63.691	66.766	73.403	76.097
50	58.164	63.167	65.030	67.505	71.420	76.154	79.490	86.662	89.564
60	68.972	74.397	76.411	79.082	83.298	88.380	91.952	99.609	102.698
70	79.715	85.527	87.680	90.531	95.023	100.425	104.215	112.319	115.582
80	90.405	96.578	98.861	101.880	106.629	112.329	116.321	124.842	128.267
90	101.054	107.565	109.969	113.145	118.136	124.117	128.300	137.211	140.789
100	111.667	118.498	121.017	124.342	129.561	135.807	140.170	149.452	153.174

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
					Confidence Level						