

**W1-2-60-1-6**

**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2016/2017**

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

**SMA 3100: THEORY OF INTEGRATION I**

**DATE: JUNE, 2017 TIME: 3 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY**

**OTHER TWO QUESTIONS**

**QUESTION ONE: 30 MARKS**

a. Define a function D on [a,b], (a,b)by

1, if

D

0, if

Where Q is the set of all rational numbers.

i. Determine the set of discontinuities of the function D (2 marks)

ii. Determine the Riemann Integrability of D (5 marks)

b. if A

then  (4 marks)

c. Let (be a measure space and suppose f: is integrable. Show

that f is finite-valued -a.e on (7 marks)

d. Let be any measurable space, were A is a-algebra of supsets of .

i. Show that any constant function on is A – measurable

(3 marks)

ii. Show that if : 1e is A- measurable then c.f is A-measurable, c (4 marks)

e. If is the Lebesgue measure space on [0,I], and g =o, gn=n for

each n in , show that = 1

**QUESTION TWO: 20 MARKS**

a. Show that if is an at most countable subset of , then

(10 marks)

Hence,

b. Show that every internal with distinct end points is uncountable.(5 marks)

c. Let -algebra of subset of X

and let E be any subset of . Show that EA

if and only if the characteristic function  defined by:-

0, if xE

 =

1, if E

is A – measurable (5 marks)

**QUESTION THREE: 20 MARKS**

a. i. Let be an extended real – valued function. Define the

functions +, - (2 marks)

ii. Let be, a measurable space and a A- measurable

function. Show that +, and - are also A-measurable

(8 marks)

b. Let be a measure space, where A is a - algebra of subsets of X. Show that if + (then

(+(

=. (10 marks)

**QUESTION FOUR: 20 MARKS**

a. State and prove Fatou’s Lemma

(you may apply M. C. T in your proof) (10 marks)

b. Let ( be the Lebergue measure space on and let

for each

Show that converges uniformity to , but that

Why does this not contradict the Monotone Convergence Theorem

(M. C.T)?

Does Faton’s Lemma apply? (10 marks)