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**W1-2-60-1-6**

**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

# University Examinations 2015/2016

# YEAR IV EXAMINATION FOR THE BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

# SMA 2306: LINEAR ALGEBRA II

**DATE: OCTOBER, 2015**  **TIME: 2 HOURS**

**INSTRUCTIONS: Answer Question ONE and Any other TWO Questions**

**Question One (30 Marks)**

a) i. Define the term a field. [1 mark]

 ii. Prove that every field is an integral domain. [6 marks]

 iii. Prove that a finite integral domain is a field. [5 marks]

b) i. What is a linear mapping? [2 marks]

ii. Let  be a linear mapping. Prove that F(0)=0 and F(-U)=-F(U) for . [4 marks]

iii. Let be the projection mapping into the x-y plane defined by  Show that is a linear mapping. [5 marks]

c) i. Distinguish between a kernel and image of a linear map. [2 marks]

 ii. Let be defined as b (iii) above, find the kernel of . [2 marks]

d) Find the determinant of the matrix . [3 marks]

**Questions Two (20 Marks)**

a) i. Define the terms rank and nullity of a linear map. [2 marks]

 ii. Let be the linear mapping defined by F(x,y,s,t)=(x-y+s+t,

 x+y+3s-3t).

 Find a basis and dimension of the image U of F. [8 marks]

b) Let the linear operator T on R2 be defined by T(x,y)=(x+2y,3x+4y). Evaluate.

1. T2
2. T3
3. F(T) where f(x)=x2-3x+4. [10 marks]

**Questions Three (20 Marks)**

a) Let  and T be the linear operator on R2 defined by T(V)=AV where V is written as column vector. Find the matrix representation of T relative to the basis B={v1=(1,3), v2=(2,5)}. [8 marks]

b) Use the co-factor expansion method to find the determinant of 

[4 marks]

c) Find the values of for which the matrix below has determinant zero. [8 marks]

**Questions Four (20 Marks)**

a) Let . Evaluate:

1. Det A2 [3 marks]
2. Det 2A [2 marks]
3. Det A-1 [1 mark]

b) Find the Eigen values and corresponding Eigen vectors of . [9 marks]

c) Let V be a vector space of real continuous functions on the interval Let be the integral operator defined by  show that is a linear functional. [5 marks]

**Questions Five (20 Marks)**

a) Let be define by the matrix . Let B be the matrix representation of A relative to the basis {(1,4),(3,0)}. Find B. [5 marks]

b) Find the trace and determinant for a linear operator on IR3 defined by T(xyz)=(2x-z,x+2y-4z, 3x-y+z). [5 marks]

 c) i. State Cayley – Hamiltons theorem. [1 mark]

 ii. Find the minimal polynomial of . [4 marks]

d) i. Define a nil potent linear operator. [1 mark]

 ii. Let . Show that A is nilpotent and compute its index.

[4 marks]