

W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2016/2017**

**YEAR II SEMESTER II EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INDUSTRIAL MATHEMATICS**

**SMA 2217: DIFFERENTIAL EQUATIONS II**

**DATE: JUNE 2017 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

1. Define the following terms;
2. Wronskian
3. Fundamental set

of n functions {y,(x), y2 (x) …….yn (x)} [4 marks]

1. Prove that y1 (x) = and y2 = , form a fundamental set of solutions to the second order o.d.e. Hence state the general solution. [6 marks]
2. State the conditions for which the total differential equation in two and three variables is integrable.

Show that x2 (y2 – a2) dx + y(x2 – z2) dy – z(y2 – a2) dz = 0 integrable and hence find its primitive. [8 marks]

1. Solve the Euler differential equation x2y11 + 3xy1 + 4y = 0 [6 marks]
2. Identify the singular points of the differential equation x(x-1)2 y11 – 3y1 + 5y = 0 and classify them. [6 marks]

**QUESTION TWO (20 MARKS)**

1. Determine if the functions f(x) = 9cos (2x) and g(x) = 2cos2 (x) – 2sin2 (x) are linearly dependant or independent. [7 marks]
2. Find the general solution to 2t 2y11 + ty1 – 3y = 0 given that the first solution is y1 (t) = t-1

[8 marks]

1. Without finding the solutions to the D.E below determine the Wronkskian of the two solutions in t4y11 – 2t3y1 – t8y = 0 [5 marks]

**QUESTION THREE (20 MARKS)**

(a)(i) The Legendre equation has the form (1 – z2)y11 – 2zy1 + L (L+1) y = 0

Where L is a constant. Show that z = 0 is an ordinary point and z = ±1 are regular

singular points of the equation. [5 marks]

(ii) Find out if the Legendre equation has an ordinary point or a regular singular point as z →

∞. [5 marks]

1. Solve the Bessel differential equation z2y11 + zy1 + (z2 – γ2) y = 0 about z = 0 [10 marks]

**QUESTION FOUR (20 MARKS)**

1. Show that the homogeneous total differential equation yzdx + 2xzdy – 3xydz = 0 is integrable. Hence find the primitive. [10 marks]
2. Given that the differential equation x1(t) = f[x(t)] with f(0) = 0 and continuous on f represents a certain physical system. Prove that if we find a Lyapunov function v(x) such that;

[x(t)] ≤ 0 for all x ≠ 0 and xϵR, then the system is locally stable. [10 marks]