MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 102: CALCULUS I

Date: 4th May, 2016

Time: 2.30 - 4.30pm

INSTRUCTIONS:

- Answer Question ONE and any other TWO questions.
- Start each question of a fresh page
- Indicate question numbers clearly at the top of each page

Question One [30marks]

a) Let f be a function defined by

$$f(x) = \begin{cases} \frac{3x}{|x|+2x^2}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

Does $\lim_{x \to 0} f(x)$ exist? [5mks]

b) A function f is defined by

$$f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{when } x \neq 0 \\ A, & \text{when } x = 0 \end{cases}$$

Find A so that f is continuous at $x = 0$. [6mks]

c) Show that, of all rectangles with a given parameter, the square has the largest area. [6mks]

d) Find the derivative of $f(x) = \frac{1}{\sqrt{x}}$ from first principle. [5mks]

e) Find derivatives of the following functions

i) $f(x) = \frac{3x^2-2}{x^2+7}$ [4mks]

ii) $f(x) = x^x$ [4mks]

Question Two [20 marks]

a) Prove that the derivative of the product $k(x) = f(x)g(x)$ of two differentiable functions f and g is given by [8mks]

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

b) If $x = a(1 - \cos\theta)$, $y = a(\theta + \sin\theta)$, show that $\frac{dy}{dx} = \cot\frac{\theta}{2}$ [7mks]

c) Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$. [5mks]

Question Three [20 marks]

a) Find the derivatives of the following functions

i) $y = \tan^{-1} e^{2x}$ [5mks]

ii) $y = \frac{1-\cos x}{1+\cos x}$ [5mks]

b) Two truck convoys leave a depot. Convoy A travelling East at $40km/h$ and convoy B travelling North at $30km/h$. How fast is the distance between the convoys changing 6 minutes later, when convoy A is $4km$ from base and convoy B is $3km$ from base? [10mks]

Question Four [20 marks]

a) Evaluate the following limits

    i) $\lim_{x \to a} \frac{x^n - a^n}{x - a}$ [4mks]

    ii) $\lim_{x \to 0} \frac{\sin 7x}{\tan 3x}$ [3mks]

    iii) $\lim_{x \to 0} \frac{x}{|x|}$ [3mks]

b) The motion of a stone thrown vertically upwards satisfies an equation of the form $s(t) = at^2 + bt$. If the maximum height reached by the stone is $4.9m$ and if its acceleration is $-9.8m/s^2$, find its height after half a second. [10mks]

Question Five [20marks]

a)   i) Find the general solution of the differential equation [2mks]

$$\frac{dy}{dx} = 3t^2$$

    ii) An evergreen nursery usually sells a certain schrub tree after 6 years of growth and shaping. The growth rate during 6 years is approximated by

$$\frac{dh}{dt} = \frac{1}{2}t + 2$$

where t is the time in years and h is the height in inches. The seedlings are 5 inches tall when planted($t=0$). How tall are the shrubs when they are sold. [6mks]

b) Sketch the graph of the function $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$ [12mks]

END