**QUESTION ONE (30MKS)**

a). Find

i. $E(X)$

ii. $E(X^2)$

iii. $E(X - X^2)$

For the probability distribution below. [6mks]

| $X$ | 8 | 12 | 16 | 20 | 24 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{12}$ |

b) Two fair dice are rolled and face values are noted.

(i) What is the probability space? [3mks]

(ii) What is the probability that the sum of the numbers showing is 7?[2mks]

(iii) What is the probability that both dice show number 2? [2mks]

c). If $P(A) = 0.24$, $P(B) = 0.67$, and $P(A \cap B) = 0.09$, find

(i) $P(A \cup B)$ [2mks]

(ii) $P((A \cup B)^c)$ [2mks]

(iii) $P(A^c \cup B^c)$ [2mks]

d). The number of births announced in the personal column of local weekly newspaper may modeled by a Poisson distribution with mean 2.4.

Find the probability that in a particular week

i) Three or fewer births will be announced [2mks]

ii) Exactly four births will be announced. [2mks]

e). Find the probability distribution of the random variable that has the probability distribution [3mks]

$$f(x) = \frac{x}{15} \qquad x = 1,2,3,4,5$$

f). In a certain state 25% of all cars emit excessive amount of pollutants. If the probability is 0.99 that a car emitting excessive a mounts of pollutants will fail, its states vehicles emission test, and the probability is 0.17 that a car not [illegible] of pollutants will nevertheless fail the test, what f). In a certain state 25% of all cars emit excessive amount of pollutants. If the probability is 0.99 that a car emitting excessive a mounts of pollutants will fail, its states vehicles emission test, and the probability is 0.17 that a car not emitting excessive amount of pollutants will nevertheless fail the test, what is the probability that a car fail test actually emits excessive amount of pollutants. **[4mks]**

## QUESTION TWO (20MKS)

Find $k$ so that the function given by $p(x) = \frac{k}{x+1}$ , $x = 1, 2, 3, 4$ is a probability function. **[3mks]**

a). (i) For $X \sim N(0, 1)$, calculate $P(Z \geq 1.13)$. **[2mks]**

(ii) For $X \sim N(5, 4)$, calculate $P(-2.5 < X < 10)$. **[3mks]**

b). The marks of 500 candidates in an examination are normally distributed with mean of 45 marks and standard deviation of 20 marks .

i. Given that the pass mark is 41, estimate the number of candidates who passed the examination **[3mks]**

ii. If 5% of that candidates obtained a distinction by scoring x marks or more estimate he value of x **[4mks]**

iii. Estimate the interquartile range of the distribution **[5mks]**

## QUESTION THREE (20MKS)

a). A discrete random variable $X$ has the cumulative distribution function

| $X$ | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| $F(x)$ | $\frac{1}{12}$ | $\frac{1}{2}$ | $\frac{5}{6}$ | 1 |

i) Write down the probability distribution of $X$ **[2mks]**

ii) Find the probability distribution of the sum of two independent observations from $X$ **[3mks]**

iii) Find the mean and variance of the distribution of the sum **[3mks]**

b) The probability density function of a random variable $X$ is given

$$f(x) = \begin{cases} cx & 0 < x < 4 \\ 0 & otherwise \end{cases}$$

i) Find $c$. **[2mks]**

ii) Find the distribution function $F(x)$. **[2mks]**

iii) Compute $P(1 < x < 3)$. **[2mks]**

c). The random variable $X$ is distributed $X \sim B(7,0.2)$. Find correct to three decimal places

i. $P(1 < X \leq 4)$ **[2mks]**

d) At selfite Supermarket, 60% of customers pay by credit card. Find the probability that in a randomly selected sample of ten customers

i. Exactly two pay by credit card **[2mks]**
ii. More than seven pay by credit card **[2mks]**

## QUESTION FOUR (20MKS)

Suppose a statistics class contains 70% male and 30% female students. It is known that in a test,5% of males and 10% of females got an "A" grade. If one student from this class is randomly selected and observed to have an "A" grade, what is the probability that this is a male student? **[3mks]**

Suppose that three types of antimissile defense systems are being tested. From the design point of view, each of these systems has an equally likely chance of detecting and destroying an incoming missile within arange of 250 miles with a speed ranging up to nine times the speed of sound. However, in actual practice it b). Let X be a continuous random variable which assumes value in the interval $(0,\infty)$. Let X has a p.d.f given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 2 \\ 0 & elsewhere \end{cases}$$

obtain the c.d.f of X

[2mks]

c) If $X$ is a uniformly distributed random variable over $(0, 10)$, calculate the probability that

(i) $X > 6$, and [2mks]

(ii) $3 < X < 8$. [2kms]

d) Let the function $f(x) = \lambda x e^{-x}$ , $x > 0$ 0, otherwise.

i) For what value of $\lambda$ is $f$ a pdf? . [2mks]

ii) Find $F(x)$. [2mks]

e) Let $Y$ be a random variable with p.d.f

$$f(x) = \begin{cases} \frac{3}{64}y^2(4-y) & 0 \le y \le 4 \\ 0 \; elsewhere \end{cases}$$

i) Find the expected value and variance of $Y$. [2mks]

ii) Let $X = 300Y + 50$. Find $E(X)$ and $Var(X)$, and. [2mks]

iii) Find $P(X > 750)$. [2mks]

has been observed that the precisions of these antimissile systems are not the same, that is, the first system will usually detect and destroy the target 10 of 12 times, the second will detect and destroy it 9 of 12 times, and the third will detect and destroy it 8 of 12 times. We have observed that a target has been detected and destroyed. What is the probability that the antimissile defense system was of the third type? [4mks]

In a tank containing 10 fishes, there are three yellow and seven black fishes. We select three fishes at random.

(a) What is the probability that exactly one yellow fish gets selected? [3mks]

(b) What is the probability that at most one yellow fish gets selected? [3mks]

(c) What is the probability that at least one yellow fish gets selected? [3mks]

To find out the prevalence of smallpox vaccine use, a researcher inquired into the number of times a randomly selected 200 people aged 15 and over in an African village had been vaccinated. He obtained the following figures: never, 17 people; once, 30; twice, 58; three times, 51; four times, 38; five times, 7. Assuming these proportions continue to hold exhaustively for the population of that village, what is the expected number of times those people in the village had been vaccinated, and what is the standard deviation? [4mks]

## QUESTION FIVE (20MKS)

a). Let X be a continuous random variable which assumes value in the interval $(0,\infty)$. Let X has a p.d.f given by

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Calculate [2mks]

i. $P(X>a)$ $a > 0$

ii. The conditional probability of the event that $X>a+c$ given $X>c$ where a and c are positive real numbers [2mks]