MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF ARTS AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 111: INTRODUCTION TO CALCULUS

Date: 7th January, 2016 Time: 8.30 - 10.30 am

INSTRUCTIONS:

- This paper consists of FIVE questions.
- Answer question ONE and any other TWO questions.
- Show all the necessary worrkings
- Observe further instructions from the booklet.

QUESTION ONE (COMPULSORY) (30mks)

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(a) (i) Define derivative of a function $f(x)$ [2mks]

(ii) Consider the function f defined by the formula

$$f(x) = \frac{x^2 - 1}{x - 1}$$

State the domain and the range, hence draw the graph of f . [3mks]

(iii) Prove that $\lim_{x \to 2} f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}$$

[4mks]

(b) A 13-foot ladder leans against a vertical wall. If the bottom of the ladder is slipping away from the base of the wall at the rate of 2 feet per second, how fast is the top of the ladder moving down the wall when the bottom of the ladder is 5 feet from the base? [5mks]

(c) (i) Prove that $\lim_{x \to 1} \frac{|x|}{x}$ does not exist. [3mks]

(ii) Show that

$$\lim_{x \to 3} 4x - 5 = 7$$

[4mks]

(d) If $g(s,t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the equation $t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$ [4mks]

(e) Find the absolute extrema, if any, of the function $f(x) = e^{(x^3 - 3x^2)}$ on the interval $(0, +\infty)$. [5mks]

QUESTION TWO (20mks)

(a) Using implicit differentiation, find y' given $\sin y + x^2y^3 - \cos x = 2y$

[5mks]

(b) Let $f(x)$ and $g(x)$ be two differentiable functions. Show that

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2};\ g(x) \neq 0$$

for all x [7mks]

(c) A red cross aircraft is dropping emergency food and medical supplies into a disaster area. If the aircraft releases the supplies immediately above the edge of an open field $700ft$ long and if the cargo moves along the path

$$x = 120t, \qquad \&\ y = -16t^2 + 500, \qquad t \geq 0$$

does the cargo land in the field? The co-ordinates x and y are measured in feet and the parameter t (time in seconds since release). Find a cartesian equation for the path of the falling cargo and the cargo's rate of descent relative to its forward motion when it hits the ground. [8mks]

QUESTION THREE (20mks)

(a) Given the function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{1}{(x+2)}, & x < -2 \\ x^2 - 5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$$

(i) $\lim_{x\to -2} f(x)$

(ii) $\lim_{x\to 0} f(x)$

(iii) $\lim_{x\to 3} f(x)$

[6mks]

(b) Use first principle to differentiate $f(x) = \sin x$ with respect to x is $\cos x$ [7mks]

(c) Explain the meaning of higher order derivatives hence find the second derivative of the function

$$f(t) = \frac{t + \sin t}{t + 2}$$

[7mks]

QUESTION FOUR (20mks)

(a) Compute the area of the surface that is enclosed by $y = e^{x}$ and $y = e^{-x}$ and the line $x = 2$ [4mks]

(b) Let I be an interval of real line and suppose that f, g and h are real valued functions defined on I such that $g(x) \leq f(x) \leq h(x)$. If $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, $a \in I$, then show that

$$\lim_{x\to a} f(x) = L$$

Given that $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2}$ for all $x \neq 0$, find $\lim_{x\to 0} u(x)$ no matter how complicated $u(x)$ is. [6mks]

QUESTION FIVE (20mks)

(a) Air is leaking out of a spherical balloon at the rate of 3 cubic inches per minute. When the radius is 5 inches, how fast is the radius decreasing? [5mks]

(b) Differentiate between a continuous and a differentiable function at a number c. [4mks]

(c) Find by logarithmic differentiation, an expression for $\frac{dy}{dx}$ in terms of x given that $y = \frac{x^2}{\sqrt{(x-1)}}$ [5mks]

(d) The equation of a curve is $y = 2x^3 - 7x + 15$, write down an expression for $\frac{dy}{dx}$ and hence find;

(i) the equation of the tangent to the curve at $(2, 3)$

(ii) the approximate change in y as x increases from 2 to 2.03, stating whether this is an increase or a decrease. [6mks]