UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

SPH 603: QUANTUM MECHANICS

DATE: APRIL 12, 2017 **TIME: 2:00-5:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

$h = 6.62 \times 10^{-34}$ J.s

$e = 1.6 \times 10^{-19}$ C

$c = 2.998 \times 10^{8}$ m.s^{-1}

$1\ u = 1.66054 \times 10^{-27}$ kg

$m_p = 1.673 \times 10^{-27}$ kg

$m_n = 1.009\ u$

$m_e = 9.11 \times 10^{-31}$ kg

QUESTION ONE (30 MARKS)

a) A particle of mass m is confined to a one-dimensional region $0 \leq x \leq a$ n as shown in Fig. 1. At $t.$ = 0 its normalized wave function is

$$\psi(x, t=0) = \sqrt{8/5a}\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]\sin(\pi x/a).$$

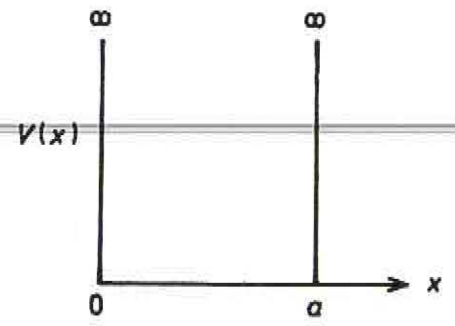

Fig. 1

i) What is the wave function at a later time $t = t_0$ (12 marks)

ii) What is the average energy of the system at $t = 0$ and at $t = t_0$? (3 marks)

iii) What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \leq x \leq a/2$) at $t = t_o$? (3 marks)

b) An electron is confined in the ground state in a one-dimensional box of width 10^{-10}m. Its energy is 38 eV. Calculate:

i) The energy of the electron in its first excited state, (2 marks)

ii) The average force on the walls of the box when the electron is in the ground state. (10 marks)

QUESTION TWO (20 MARKS)

a) Consider an electron moving in a spherically symmetric potential $V = kr$, where $k > 0$.

i) Use the uncertainty principle to estimate the ground state energy. (10 marks)

ii) Use the Bohr Somerfield quantization rule to calculate the ground state energy. (10 marks)

QUESTION THREE (20 MARKS)

a) You are given a real operator $\hat{A}$ satisfying the quadratic equation $\hat{A}^2 - 3\hat{A} + 2 = 0$. This is the lowest-order equation that $\hat{A}$ obeys.

i) What are the eigenvalucs of $\hat{A}$?

ii) What are the eigenstates of $\hat{A}$?

iii) Prove that $\hat{A}$ is an observable. (10 marks)

b) The three matrix operators for spin one satisfy $S_xS_y - S_yS_x = iS_z$, and cyclic permutations. Show that, $s_z^3 = s_z, \quad (s_x \pm i s_y)^3 = 0.$ (5 marks)

c) The spin functions for a free electron in a basis where $\hat{s}_z$, is diagonal can be written as $\binom{1}{0}$ and $\binom{0}{1}$ with eigenvalues of $\hat{s}_z$, being +1/2 and -1/2 respectively. Using this basis find a normalized eigenfunction of $\hat{s}_y$, with eigenvalue - 1/2. (5 marks)

QUESTION FOUR (20 MARKS)

a) Derive the quantum mechanical expression for the s-wave cross section for scattering from a hard sphere of radius *R*. (10 marks)

b) The range of the potential between two hydrogen atoms is approximately 4Å. For a gas in thermal equilibrium, obtain a numerical estimate of the temperature below which the atom-atom scattering is essentially s-wave. (10 marks)

QUESTION FIVE (20 MARKS)

a) Derive the condition for the validity of the WKB approximation for the one-dimensional time-independent Schrödinger equation, and show that the approximation must fail in immediate neighbourhood of a classical turning point. (15 marks)

b) Explain, using perturbation theory, why the ground state energy of an atom always decreases when the atom is placed in an external electric field. (5 marks)

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