UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

SPH 601: CLASSICAL MECHANICS

DATE: APRIL 10, 2017 **TIME: 2:00-5:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: : Unless otherwise specified, take;

- Gravitational acceleration, g = 9.8 $m.s^{-2}$
- Speed of light , c = 3.0×10^8 $m.s^{-1}$
- Gravitational constant, G = $6.67 X 10^{-11}$ m^3/s^2. kg. (or G = $6.67 X 10^{-11} Nm^2/kg^2$)
- Earth's mass, M=$5.98 X 10^{24}$ kg.
- Earth's radius, R_E = 6.37 X 106 m.
- Density of the *earth,* $\rho = 5.51 X 10^3$ kgm^{-3}

QUESTION ONE (30 MARKS)

a) Let $\vec{r}$ be the radius vector of a particle of mass m from a given origin. If the vector velocity $\vec{v}$, is given by $\vec{v} = \frac{d\vec{r}}{dt}$, stating all assumptions and the conservation theorems show that if momentum is conserved then $\vec{N} = \frac{d\vec{L}}{dt}$, where $\vec{N}$ and $\vec{L}$ have their usual meaning. (8 marks)

b) A ladder of length L and mass M has its bottom end attached to the ground by a pivot. It makes an angle θ with the horizontal, and is held up by a mass less stick of length ` which is also attached to the ground by a pivot (see figure 1). The ladder and the stick are perpendicular to each other. Find the force that the stick exerts on the ladder. (6 marks)

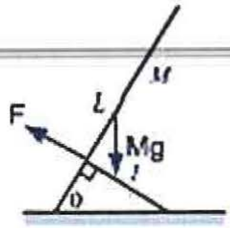

Fig. 1

c) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy: $\frac{dT}{dt} = \boldsymbol{F}.\boldsymbol{v}$, while if the mass varies with time the corresponding equation is; $\frac{d(mT)}{dt} = \boldsymbol{F}.\boldsymbol{p}$ (6 marks)

d) Show that $\vec{\dot{P}} = \vec{F}^e$ which gives the conservation theorem for the linear momentum of a system of particles. (6 marks)

e) If a particle is subjected to a central force only, then its angular momentum is conserved. That is, If $V(\mathrm{r}) = V(r)$; , then $\frac{dL}{dt} = 0$. Prove this. (4 marks)

QUESTION TWO (20 MARKS)

a) Consider a particle of mass m moving in a two-dimensional central force, $\vec{F} = \lambda\vec{r}$ where $\vec{r} = x\mathrm{e}_x + y\mathrm{e}_y$ and λ is a positive constant. At an initial time t = 0, its position is $\vec{r}_0 = a\mathrm{e}_x + b\mathrm{e}_y$ where a and b are positive constants. The initial velocity is not known. However, the product of velocity components does not depend on time and equals to a non-zero constant all time. (That is $v_x(t)v_y(t)=$non-zero constant). Show that the particle moves in a hyperbola (that means $x(t)y(t)=$constant). (10 marks)

b) A particle moves in a central force field given by the potential $V = -k\frac{e^{-ar}}{r}$, where k and a are positive constants. Using the method of the equivalent one –dimensional potential discuss the nature of the motion stating the ranges of l and E appropriate to each type of motion. When are circular orbits possible? Find the period of small radial oscillations about the circular motion. (10 marks)

QUESTION THREE (20 MARKS)

Two springs have spring constants, k_1 and k_2, respectively and one end of each spring is attached to a separate wall as shown in Figure. A ball of mass m connects the two springs. The ball can oscillate only horizontally. A mass less rigid rod of length L is attached to the ball and is free to rotate around the ball (that is, the angle θ, can vary from $-\pi$ to $+\pi$). Another ball of mass M is attached to the other end of the rod. The position x of m is measured from the equilibrium position of the springs and the coordinate θ is measured from the vertical.

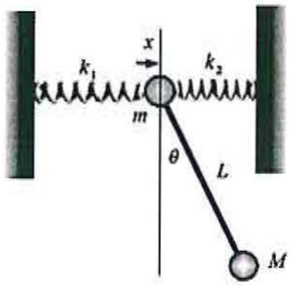

Fig. 2

a) Find the Lagrangian of the system and the Euler-Lagrange equations for each coordinate. (10 marks)

b) When m is negligibly small compared to M and the amplitude of the oscillation is small, show that

$$\theta \approx \frac{k_1 + k_2}{Mg} x.$$

(5 marks)

c) Using the above approximation, find the frequency of the small amplitude oscillation.

(5 marks)

QUESTION FOUR (20 MARKS)

Two particles move about each other in circular orbits under the influence of gravitational forces with a period τ. Their motion is suddenly stopped and are then released and allowed to fall into each other. Prove that they collide after time $\tau/4\sqrt{2}$. (20 marks)

QUESTION FIVE (20 MARKS)

a) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle, then the force varies as the inverse 5^{th} power of distance. (8 marks)

b) Show that for the orbit described, the total energy of the particle is zero. (6 marks)

c) Find the period of the motion. (6 marks)

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