UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR
SECOND SEMESTER EXAMINATION

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 403: SOLID STATE PHYSICS II

DATE: APRIL 6, 2017 **TIME: 2:00-4:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

- Gravitational acceleration, $g = 9.8\ m.s^{-2}$
- Speed of light, $c = 3.0 \times 10^8\ m.s^{-1}$
- Gravitational constant, $G = 6.67 \times 10^{-11}\ m^3/s^2.kg$.
- Earth's mass, $M = 5.98 \times 10^{24}$ kg.
- Earth's radius, $R_E = 6.37 \times 10^6$ m.

Mass of the electron $m = 9.11 \times 10^{-31}$ kg
Planck's constant $h = 2\pi \times 1.05 \times 10^{-34}$ Js
elementary charge $e = 1.60 \times 10^{-19}$ C
one electron volt $= 1.60 \times 10^{-19}$ J
Boltzmann's constant $k_B = 1.38 \times 10^{-23}\ JK^{-1}$
permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$
Avogadro's number $= 6.02 \times 10^{23}$

QUESTION ONE (30 MARKS)

a) Consider the effect of anharmonic terms in the potential energy on the separation of a pair of atoms at temperature T. Taking the potential energy of the atoms at a displacement x from their equilibrium separation at 0°K as $V(x) = cx^2 - gx^3 - fx^4$ and explaining the meaning of the terms, show that the average displacement is given by

$$\bar{x} = 3kTg/4c^2$$

(5 marks)

b) Einstein's model of solids gives the expression for the specific heat as

$$C_v = 3N_0k\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2} \qquad \text{where } \theta_E = h\nu_E/k.$$

The factor θ_E is called the characteristic temperature. Show that

i) At high temperatures Dulong Petit law is reproduced.

ii) But at very low temperatures the T^3 law is not given (5 marks)

c) The density of states function for electrons in a metal is given by: $Z(E)\mathrm{d}E = 13.6 \times 10^{27} E^{1/2}\mathrm{d}E$. Calculate the Fermi level at a temperature few degrees above absolute zero for copper which has 8.5×10^{28} electrons per cubic metre and hence find the velocity of electrons at the Fermi level in copper. (5 marks)

d) i) Calculate the separations of the sets of planes which produce strong x-ray diffractions beams at angles 4° and 8° in the first order, given that the x-ray wavelength is 0.1 nm. (3 marks)

ii) At what angle will a diffracted beam emerge from the (111) planes of a face centered cubic (FCC) crystal of unit cell length 0.4 nm? Assume diffraction occurs in the first order and that the x-ray wavelength is 0.3 nm. (4 marks)

iii) An x-ray beam of wavelength 0.16 nm is incident on a set of planes of a certain crystal. The first Bragg reflection is observed for an incidence angle of 36°.
What is the plane separation? Will there be any higher order reflections? (3 marks)

e) The density of nickel is 8.90×10^3 kg/m^3. If the Atomic weight of Ni is 58.71 gm/mol. Calculate (i) the saturation magnetization (ii) the saturation flux density. (5 marks)

QUESTION TWO (20 MARKS)

Atoms in crystals vibrate naturally around their equilibrium lattice positions because of temperature. Consider the spring model of a linear lattice where the force F_s exerted on the s^{th} atom by the $(s+1)^{th}$ atom is always proportional to their relative atomic displacements $q_{s+1} - q_s$. If a wave is introduced into the crystal with lattice points are so densely packed that it can be considered as a continuous medium.

i) Stating all assumptions and approximations, derive the equation of motion for the one dimensional lattice and give the expression for the wave velocity. (7 marks)

ii) Show that the dispersion relation is given by the equation

$$\omega^2 = \frac{2}{m} \sum_{n>0} C_n (1 - \cos(nka)).$$

Also, stating all approximations show the changes in the dispersion relation under the long wavelength and nearest neighbour approximations respectively. (7 marks)

iii) Show that the force constant between any two atoms in the linear lattice is given by the relation

$$C_\gamma = -\frac{ma}{2\pi}\int_{-\pi/a}^{\pi/a} \omega^2 \cos(\gamma ka)\cdot dk,$$

(6 marks)

QUESTION THREE (20 MARKS)

a) Show that the classical average internal energy of a one-dimensional harmonic oscillator in thermal equilibrium at temperature T is kT where k is the Boltzmann constant.

(5 marks)

b) Derive the expression for the free energy $F = U + TS$ of a collection of quantum harmonic oscillators. (15 marks)

QUESTION FOUR (20 MARKS)

a) State the essential composition of Drude's model. (5 marks)

b) For a free electron gas in a metal, the number of states per unit volume with energies from E to $E + \mathrm{d}E$ is given by

$$n(E)\mathrm{d}E = \frac{2\pi}{h^3}(2m)^{3/2}E^{1/2}\mathrm{d}E$$

Show that the total energy is, $\frac{3}{5}NE_{max}$ (10 marks)

c) The Fermi energy in gold is 5.54 eV (i) calculate the average energy of the free electrons in gold at 0°K. (ii) Find the corresponding speed of free electrons (iii) What temperature is necessary for the average kinetic energy of gas molecules to posses this value?

(5 marks)

QUESTION FIVE (20 MARKS)

a) Using the equation $$U = \frac{L}{\pi v_0}\int_0^{\omega_m} \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}\, d\omega.$$

i) Show that the heat capacity of a monatomic lattice in Debye approximation is proportional to T/Θ for low temperatures such that, $T \ll \Theta$, where Θ is the effective Debye temperature in one dimension. (10 marks)

ii) Show that at high temperatures Debye's model gives Dulong Petit law and at low temperatures it gives $C_V \propto T^3$ in agreement with the experiment. (10 marks)

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