UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 309: QUANTUM MECHANICS I

DATE: APRIL 11, 2018 **TIME: 2:00-4:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

K_B =1.38066× 1010^{-23}J/K

$h = 6.62 \times 10^{-34}$ J.s

$e = 1.6 \times 10^{-19}$ C

$c = 2.998 \times 10^{8}$ m.s^{-1}

$1\ u = 1.66054 \times 10^{-27}$ kg

$m_p = 1.673 \times 10^{-27}$ kg

$m_n = 1.009\ u$

$m_e = 9.11 \times 10^{-31}$ kg

QUESTION ONE (30 MARKS)

a) Calculate the de Broglie wavelength associated with an electron which has a kinetic energy of 15 KeV. (3 marks)

b) Calculate the wavelength in *nm* of electrons which have been accelerated from rest through a potential difference of 54V. (3 marks)

c) Show that the de Broglie wavelength for neutrons is given by $\lambda = 0.286\,\text{Å}/\sqrt{E}$, where E is in electron-volts (eV). (3 marks)

d) State Heisenberg's Uncertainty principle in words. (2 marks)

e) Explain briefly how Wheeler's thought experiment illustrates the concepts of measurement and uncertainty to provide a logical and consistent description of the wave-particle properties of quantum particles. (4 marks)

f) Calculate the minimum wavelength of the radiation emitted by an X-ray tube operated at 30 kV. (3 marks)

g) If the minimum wavelength recorded in the continuous X-ray spectrum from a 50 kV tube is 0.247A°, calculate the value of Plank's constant. (3 marks)

h) Calculate the maximum change in the wavelength of Compton scattered radiation. (3 marks)

i) Quantum phenomena are often negligible in the "macroscopic world". Show this numerically for the case of the diffraction of a tennis ball of mass $m = 0.1\ kg$ moving at speed $v = 0.5\ m/s$ by a window of size $1x1.5\text{m}^2$. (3marks)

j) If $\psi(x) = \frac{N}{x^2+a^2}$, calculate the normalization constant N (3marks)

QUESTION TWO (20 MARKS)

a) In a nuclear reactor, neutrons are brought into thermal equilibrium by repeated collisions in heavy water at $T = 300$ K. Calculate the average energy (in eV) and the typical wavelength of the neutrons. Explain why they are diffracted when they pass through a crystalline solid. (6 marks)

b) A free particle with momentum p and energy E is conventionally represented by the wave function

$$\Psi(x, t) = A\,e^{+i(px-Et)/\hbar}.$$

Scientists may have chosen to represent the free particle with momentum p and energy E by the wave function

$$\Psi(x, t) = A\,e^{-i(px-Et)/\hbar}.$$

What is the form of the Schrödinger equation for this case? (4 marks)

c) Use the uncertainty relation to find an estimate of the ground-state energy of a harmonic oscillator. (10 marks)

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 .$$

QUESTION THREE (20 MARKS)

a) Describe how the Heisenberg microscope can be used to explain the wave-particle duality of quantum particles by allowing for the role of the uncertainty principle in the act of measurement. (10 marks)

b) A wave packet is represented by $\Psi(x,t) = \int_{k-\Delta k}^{k+\Delta k} A\cos(k'x - \omega' t)\,\mathrm{d}k'$ where A is a constant. Integrate over k' and simplify. Assume that the dispersion relation is $\omega' = ck'$, where c is a constant. (5 marks)

c) Prove that the real functions; $\Psi = A\cos(kx - \omega t)$ and $\Psi = A\sin(kx - \omega t)$ are **NOT** solutions of the Schrödinger equation for a free particle. (5 marks)

QUESTION FOUR (20 MARKS)

a) Consider a Gaussian distribution, with standard deviation σ, given by

$$\rho(x)\mathrm{d}x = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-x^2/2\sigma^2}\mathrm{d}x \qquad \text{with } -\infty < x < +\infty.$$

This probability distribution is normalized because

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-x^2/2\sigma^2}\,\mathrm{d}x = 1.$$

(i) By considering the transformation $x \to -x$ on the function $x\rho(x)$, find the expectation value of x. (4 marks)

(ii) Given that,

$$\int_{-\infty}^{+\infty} \rho(x)\,\mathrm{d}x = \Big[x\,\rho(x)\Big]_{-\infty}^{+\infty} - \int_{-\infty}^{\infty} x\,\frac{\mathrm{d}\rho}{\mathrm{d}x}\,\mathrm{d}x,$$

show that the expectation value of x^2 is equal to σ^2. Hence verify that the standard deviation of x is equal to σ. (6 marks)

b) An unstable particle has a lifetime which obeys the exponential probability distribution,

$p(t)\,\mathrm{d}t = e^{-\lambda t}\,\lambda\,\mathrm{d}t,$ where λ is a positive decay constant.

(i) Evaluate the probability with which the particle eventually decays. (2 marks)

(ii) Determine an expression for the mean lifetime of the particle. (6 marks)

(iii) Find an expression for the probability that the particle lives for at least time T. (2 marks)

QUESTION FIVE (20 MARKS)

Solve the Time-Independent Schrodinger Equation with appropriate boundary conditions for the finite square well centred at the origin shown in Fig 5.1,

$$V(x) = \begin{cases} V_O, & x < -\frac{L}{2} \\ 0, & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ V_O, & x > \frac{L}{2} \end{cases}$$

$V > 0$

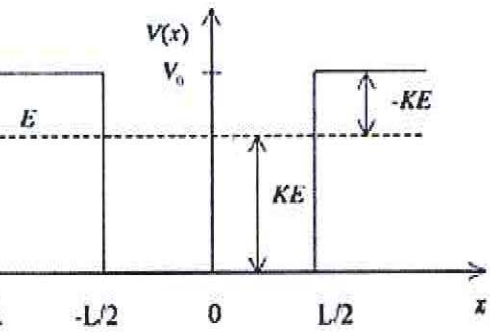

Fig 5.1

(20 marks)

--END--