UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR MAIN EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 309: QUANTUM MECHANICS I

DATE: APRIL 11, 2017 **TIME: 11:00AM-1:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

$h = 6.62 \times 10^{-34}$ J.s

$e = 1.6 \times 10^{-19}$ C

$c = 2.998 \times 10^{8}$ m.s^{-1}

$1\ u = 1.66054 \times 10^{-27}$ kg

$m_p = 1.673 \times 10^{-27}$ kg

$m_n = 1.009\ u$

$m_e = 9.11 \times 10^{-31}$ kg

$\kappa_B = 1.38066 \times 10^{-23}$ J/K

QUESTION ONE (30 MARKS)

a) Neutrons from a nuclear reactor are brought into thermal equilibrium by repeated collisions in heavy water at T =300 K. What is the average energy (in eV) and the typical wavelength of the neutrons? Explain why they are diffracted when they pass through a crystalline solid. (4 marks)

b) According to Stefan-Boltzmann's law for the radiation from a black body, the intensity of radiation, in units of Js^{-1} m^{-2}, from a body at temperature T is

$I = \sigma T^4$, where σ is Stefan-Boltzmann's constant. Because black-body radiation can be considered to be a gas of photons, i.e. quantum particles which move with velocity c with typical energies of the order of kT, the intensity I is a function of h, c and kT. Use dimensional analysis to confirm that I is proportional to T^4 and find the dependence of σ on h and c. (6 marks)

c) State and briefly explain the properties of the operators that have to represent observables in quantum mechanics. (6 marks)

d) In quantum mechanics it is the convention to represent a free particle with momentum p and energy E by the wave function

$$\Psi(x,t) = A\,e^{+i(px-Et)/\hbar}.$$

Physicists on another planet may have chosen the convention of representing a free particle with momentum p and energy E by the wave function

$$\Psi(x,t) = A\,e^{-i(px-Et)/\hbar}.$$

What is the form of the Schrödinger equation on this planet? (4 marks)

QUESTION TWO (20 MARKS)

a) Consider the Heisenberg microscope. Give a consistent description of how the wave-particle properties of quantum particles emerge by allowing for the role of the uncertainty principle in the act of measurement. (10 marks)

b) Consider the wave packet represented by $\Psi(x,t) = \int_{k-\Delta k}^{k+\Delta k} A\cos(k'x - \omega' t)\,dk'$ where A is a constant and assume that the dispersion relation is $\omega' = ck'$, where c is a constant. By integrating over k', show that $\Psi(x,t) = S(x-ct)\cos k(x-ct)$ where

$$S(x-ct) = 2A\Delta k\frac{\sin[\Delta k(x-ct)]}{[\Delta k(x-ct)]}.$$

(5 marks)

c) Verify by direct substitution that the real functions;

$\Psi = A\cos(kx-\omega t)$ and $\Psi = A\sin(kx-\omega t)$ are not solutions of the Schrödinger equation for a free particle. (5 marks)

QUESTION THREE (20 MARKS)

a) Consider a Gaussian distribution, with standard deviation σ, given by

$$\rho(x)dx = \frac{1}{\sqrt{2\pi\sigma^2}}\,e^{-x^2/2\sigma^2}dx \qquad \text{with } -\infty < x < +\infty.$$

This probability distribution is normalized because

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-x^2/2\sigma^2}\, dx = 1.$$

i) By considering the effect of the transformation $x \to -x$ on the function $xp(x)$, show that the expectation value of x is equal to zero. (4 marks)

ii) By using

$$\int_{-\infty}^{+\infty} \rho(x)\, dx = \Big[x\, \rho(x)\Big]_{-\infty}^{+\infty} - \int_{-\infty}^{\infty} x \frac{d\rho}{dx}\, dx,$$

show that the expectation value of x^2 is equal to σ^2. Hence verify that the standard deviation of x is equal to σ. (6 marks)

b) The lifetime of an unstable particle is governed by the exponential probability distribution.

$p(t)\, dt = e^{-\lambda t}\, \lambda\, dt,$ where λ is a positive decay constant.

i) Show that the probability that the particle eventually decays is equal to one. (2 marks)

ii) Find an expression for the mean lifetime of the particle. (6 marks)

iii) Find an expression for the probability that the particle lives for at least time T. (2 marks)

QUESTION FOUR (20 MARKS)

a) Consider a quantum particle in the potential energy well shown in figure 1 below.

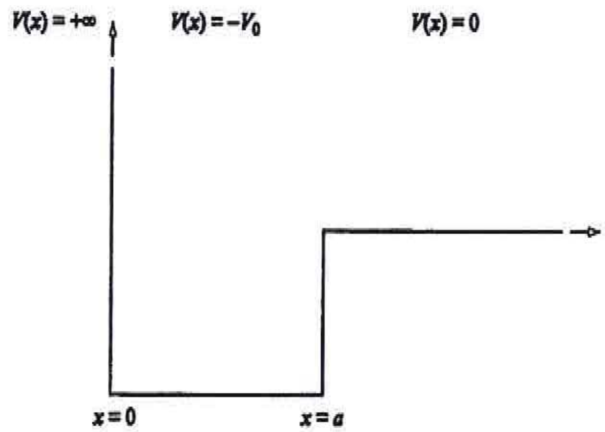

Fig.1

The Schrödinger equation for this well can be written in the form:

$$\frac{d^2}{dx^2}\psi = -k_0^2\psi\ ,\ 0 < x < a\ ;$$
$$\frac{d^2}{dx^2}\psi = \alpha^2\psi\ ,\ a < x < +\infty\ .$$

i) ~~Show that α is defined in terms of the energy of the quantum particle as~~

$$E = -\frac{\hbar^2\alpha^2}{2m}\ .$$

ii) Assume that k_0 is given by

$$E = \frac{\hbar^2 k_0^2}{2m} - V_0\ .$$

Show how the allowed wave functions and energies for this potential well can be obtained through a graphical solution. (15 marks)

b) Briefly explain the general properties of wave functions in a potential energy field with reference to bound or unbound states as well as classically allowed or forbidden regions. (5 marks)

QUESTION FIVE (20 MARKS)

a) Use the uncertainty relation to find an estimate of the ground-state energy of a harmonic oscillator.

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2\ .$$

(8 marks)

b) Discuss 4 important properties related to the time-dependence of quantum states as well as the results of measurements performed on the associated physical systems. (6 marks)

c) Explain qualitatively how different beams of atoms are sorted out in direction depending on the magnetic moment of the atom in the Stern-Gerlach experiment. Make use of diagrams where necessary. (6 marks)

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