UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 305: CLASSICAL MECHANICS

DATE: DECEMBER 2, 2016 **TIME: 2:00-4:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: : Unless otherwise specified, take;

- Gravitational acceleration, $g = 9.8\ m.s^{-2}$
- Speed of light , $c = 3.0 \times 10^8\ m.s^{-1}$
- Gravitational constant, $G = 6.67 \times 10^{-11}\ m^3/s^2.\ kg$. (or $G = 6.67 \times 10^{-11} Nm^2/kg^2$)
- Earth's mass, $M=5.98 \times 10^{24}$ kg.
- Earth's radius, $R_E = 6.37 \times 10^6$ m.
- Density of the *earth*, $\rho = 5.51 \times 10^3\ kgm^{-3}$

QUESTION ONE (30 MARKS)

a) State the three Newton's laws of motion. (3 marks)

b) Let $\vec{r}$ be the radius vector of a particle of mass m from a given origin. If the vector velocity $\vec{v}$, is given by $\vec{v} = \frac{d\vec{r}}{dt}$, stating all assumptions and the conservation theorems show that if momentum is conserved then $\vec{N} = \frac{d\vec{L}}{dt}$, where $\vec{N}$ and $\vec{L}$ have their usual meaning. (8 marks)

c) i) State the Parallel Axis Theorem. (2 marks)

ii) Verify the parallel-axis theorem for a stick of mass m and length l (5 marks)

d) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy: $\frac{dT}{dt} = \boldsymbol{F}.\boldsymbol{v}$, while if the mass varies with time the corresponding equation is; $\frac{d(mT)}{dt} = \boldsymbol{F}.\boldsymbol{p}$ (5 marks)

e) i) State the Hamilton's Principle (2 marks)

ii) Derive the Hamiltonian for a single particle of mass m moving in one dimension subject to a conservative force with a potential $U(x)$. (5 marks)

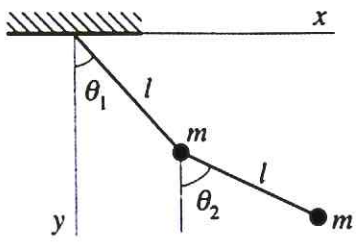
QUESTION TWO (20 MARKS)

a) A mass hangs from a mass less string of length l. Conditions have been set up so that the mass swings around in a horizontal circle with the string making an angle θ with the vertical. What is the angular frequency, ω of this motion? Under what condition does this reduce to the frequency of a plane pendulum? (10 marks)

b) Consider a block of soft wood on a horizontal frictionless surface. A compression spring with spring constant k and uncompressed length l connects the block to a wall. The block has mass M, and the spring has negligible mass. At $t = 0$ a gun fires a bullet of mass m and speed v_0 into the block, which moves back at initial speed V_i due to the impulse. Find the speed of the bullet. (10 marks)

QUESTION THREE (20 MARKS)

a) A double pendulum consists of a mass suspended by a mass less string of length l from which is suspended another such string (see Figure).



i) Write the Lagrangian of the system for $\theta_1, \theta_2 \ll 1$ (10 marks)

ii) Derive the equations of motion (5 marks)

b) A one-dimensional harmonic oscillator has Hamiltonian $H = \frac{1}{2} P^2 + \frac{1}{2} \omega^2 q^2$

Write down Hamiltonian's equation and find the general solution. (5 marks)

QUESTION FOUR (20 MARKS)

a) i) Show that the group velocity associated with a free non-relativistic particle is the classical velocity of the particle. (5 marks)

ii) For a relativistic particle with rest mass *m* and moving with the velocity of light c, the relation for total energy *E* and momentum *p* for a relativistic particle is given by $E^2 = c^2p^2 + m^2c^4$. Show that the product of group velocity v_g and the phase velocity v_p, $v_g v_p = c^2$.

(5 marks)

b) Two particles of masses m_1 and m_2 are connected by a rigid massless rod of length *r* to constitute a dumbbell which is free to move in a plane. Show that the moment of inertia of the dumbbell about an axis perpendicular to the plane passing through the centre of mass is ur^2 where u is the reduced mass. (5 marks)

c) A rod of length *L* has a non-uniform density. The mass per unit length of the rod, λ, varies as $\lambda = \lambda_o(x/L)$, where λ_o is a constant and *x* is the distance from one end. Find the center of mass of the rod. (5 marks)

QUESTION FIVE (20 MARKS)

a) A *central force* is defined to be a force that points radially and whose magnitude depends only on *r*. That is, F(r) = *F(r)*ˆr. Show that a central force is a conservative force by explicitly showing that $\nabla \times \mathbf{F} = 0$. (6 marks)

b) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle, then the force varies as the inverse fifth power of distance. (8 marks)

c) If a particle is subjected to a central force only, then its angular momentum is conserved.

That is, If $V(\mathbf{r}) = V(r)$; , then $\frac{dL}{dt} = 0$ .Prove this. (6 marks)

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