UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 304: ELECTRODYNAMICS I

DATE: NOVEMBER 29, 2016 **TIME: 11:00-1:00**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants:

- Gravitational acceleration, g = 9.8 $m.s^{-2}$
- Speed of light , c = 3.0 x 10^8 $m.s^{-1}$
- Permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ $H.m^{-1}$
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

QUESTION ONE (30 MARKS)

a) A point charge Q is placed at the origin, and a second point charge $-Q$ is placed at position $(3\ell; 0; 0)$ on the x-axis. Calculate the electric field at any point on the z-axis. Comment on the case when $z \gg \ell$. (5 marks)

b) The electric field in a region, in spherical coordinates, is $\vec{E} = 2kr^3\hat{r}$. Calculate the divergence of this field, and hence obtain the charge density in the region. (5 marks)

c) A wire with constant linear charge density λ has the shape of a semi-circle of radius a around the origin in the XY-plane. Calculate the z-component of the electric field at any point on the z-axis. (5 marks)

d) If a dielectric has polarization $\vec{P}$, the amount of charge that moved through a small surface $d\vec{a}$ as the dipoles were formed is given by $dq = \vec{P} \cdot d\vec{a}$. Use this to show that the polarized dielectric has equivalent bound charge densities given by $\sigma_b = \vec{P} \cdot \hat{n}$ and $\rho_b = -\nabla \cdot \vec{P}$. (5 marks).

e) A cube lies with one corner at the origin and the diagonally opposite corner at $(1; 1; 1)$. If the cube is made from dielectric material and has constant polarization $\vec{P} = P\hat{z}$, determine the equivalent bound charge distribution. (4 marks)

f) A sphere of radius a has polarization $\vec{P} = kr\hat{r}$. Find the equivalent bound charge densities, and hence the electric field. (6 marks)

QUESTION TWO (20 MARKS)

a) Prove that the divergence of the magnetic field, given by $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\prime) \times \hat{R}}{R^2} d\tau'$, is zero. Give reasons for your steps. [You may assume $\nabla\left(\frac{1}{R}\right) = -\frac{\hat{R}}{R^2}$.] (5 marks)

b) Give a similar expression to $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\prime) \times \hat{R}}{R^2} d\tau'$, but appropriate for line currents. Use this to derive a formula for the magnetic field at the centre of a circular ring of current i flowing clockwise and having radius a. State the direction of the magnetic field if the current loop is in the plane of the page. (5 marks)

c) A hollow metal tube (radius a) has negligible thickness and a constant current flows along the tube (parallel to its axis). The current is distributed symmetrically over the tube. If the magnetic field lines form circles around the axis of the tube, make use of Ampere's law to calculate the magnetic field everywhere. (5 marks)

d) Consider a solenoid having n loops per unit length each carrying a current i. Assuming the magnetic field of a solenoid is parallel to its axis (end-effects can be ignored), describe a suitable Amperian path and use it to show that the difference in magnitude of the magnetic field inside and outside the solenoid is $\Delta B = \mu_0 ni$. Include a diagram with your answer.

(5 marks)

QUESTION THREE (20 MARKS)

a) For a stationary circuit the emf ($\varepsilon = \oint \vec{E} \cdot d\vec{l}$) is found to correspond experimentally to the rate of change of magnetic flux ($\varepsilon = -\frac{d\Phi_B}{dt}$), where $\Phi_B = \iint \vec{B} \cdot d\vec{a}$. Use Stokes theorem to show that $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$. (4 marks)

b) In magnetostatics one can show that $\nabla \times \vec{B} = \mu_0 \vec{J}$..

i) By taking the divergence on both sides, show that this leads to $\nabla \cdot \vec{J} = 0$. (3 mark)

ii) The current flowing out through a closed surface S is given by $i = \oiint_S \vec{J} \cdot d\vec{a}$. If charge is conserved, this must correspond to the rate of decrease of the amount of charge inside the closed surface. Show that mathematically this can be expressed as $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$. (3 marks)

iii) This means the equation $\nabla \times \vec{B} = \mu_0 \vec{J}$ can only be true as long as $\frac{\partial \rho}{\partial t} = 0$, which is not always the case. Show that if instead $\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$, then taking the divergence on both sides leads to no such problem. (2 marks)

c) The electrostatic energy of a system is given by $U = \frac{1}{2} \iiint \rho V d\tau$.

i) Show that the energy of a charge Q spread evenly over a spherical surface at constant potential V is $U = \frac{1}{2} QV$. (2 marks)

ii)) For a spherical surface of radius a, with uniform charge density, the potential on the surface is $V = \frac{Q}{4\pi\varepsilon_0 a}$. If a soap bubble of radius 5 cm is initially at a potential of 200 V, determine the charge on the soap bubble. (2 marks)

iii) If the bubble then contracts to a radius of 1 cm without losing any charge, by how much does its electrostatic energy change (give your answer in J) (4 marks)

QUESTION FOUR (20 MARKS)

a) A uniform external field $\mathbf{B}_0$ induces a uniform magnetization inside a simple magnetic sphere of radius a and permeability μ. Find the magnetic moment **m** of the sphere.

(10 marks)

b) An infinite solenoid *(n* turns per unit length, current I) is filled with linear material of susceptibility *Xm.* Find the magnetic field inside the solenoid. (10 marks)

QUESTION FIVE (20 MARKS)

a) A permanent magnetic dipole, **m**, is brought up to the plane interface between vacuum, $\mu_r=1$, and a superconductor, $\mu_r=0$. The dipole is located a distance z in front of the interface.

i) With the aid of a diagram, show that the image magnetic charge induced in the superconductor by the magnetic charge q_m a distance z in front of the interface is equal to q_m and is located a distance z behind the interface. The image charge is required in order to satisfy the condition div**B**=0 and also the condition **B**=0 in the superconductor.

(5 marks)

ii) Using the results of part (a) and a diagram, calculate the force exerted on a magnetic dipole by its image when the dipole is oriented parallel with the interface. (10 marks)

iii) Calculate the force on the dipole when it is oriented normal to the interface. (5 marks)

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