UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (SCIENCE)

SPH 204: MATHEMATICAL PHYSICS 1

DATE: APRIL 4, 2018 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) For a finite series 2+7+12+17+, …Determine;

i) The 9^{th} and the 16^{th} term (3 marks)

ii) The sum of the first ten terms (3 marks)

b) The following is an infinite series;

$$\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}$$

i) Determine the sequence for its partial sums to the fourth term (3 marks)

ii) Does the series converge or diverge? If it converges, determine the sum or the value. (3 marks)

c) Determine $\int \frac{-5}{9\sqrt[4]{t^3}}dt$ (3 marks)

d) Show that $\int u dv = uv - \int v du$ can be developed from the product rule, the symbols have their usual meaning. (3 marks)

e) Differentiate between vector field and scalar field (2 marks)

f) If $\vec{A} = 2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ are two sides of a parallelogram, calculate the area enclosed by the parallelogram if the measurements are in meters (3 marks)

g) Give the order and the general solution for the following differential equation (2 marks)

$$\frac{dy}{dx} = 3$$

h) Identify the general difference between Laurent and Taylor series (1 mark)

i) Highlight how del operator can be used to determine gradient (2 marks)

j) Deduce any two properties of line integrals (2 marks)

QUESTION TWO (20 MARKS)

a) Derive the Maclaurin's theorem and state three conditions of its relevance. (8 marks)

b) Using the theorem in (a) above, find the first five non-zero terms for the function $f(x) = \sin x$ (7 marks)

c) Evaluate the following using Maclaurin's theorem, correct to 3 significant figures or at fourth non-zero term. (5 marks)

$$\int_0^1 \frac{\sin\theta}{\theta}\,d\theta$$

QUESTION THREE (20 MARKS)

a) Highlight four laws applied in vector algebra. (4 marks)

b) Using the concept of dot product, determine the angle between the two vectors $\vec{F} = 2\hat{\imath} + 5\hat{\jmath} - \hat{k}$ and $\vec{G} = 3\hat{\jmath} + \hat{k}$ (6 marks)

c) By unit vector approach, show that the cross product of any two non-zero vectors is a scalar. (4 marks)

d) For $\vec{A} = 2\hat{k} + 2\hat{\jmath} - \hat{\imath}$, $\vec{B} = 6\hat{\imath} + 3\hat{\jmath} - \hat{k}$, and $\vec{C} = 3\hat{\imath} - 2\hat{k}$, show that $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Define a differential equation (1 mark)

b) Given the first order differential equation $5\frac{dy}{dx} + 2x = 3$, determine;

i) The general solution by separation of variables (4 marks)

ii) The particular solution given that $y = 1\frac{2}{5}$ when $x = 2$ (3 marks)

c) Solve the homogenous first order differential equation $y - x = x\frac{dy}{dx}$, given that $x = 1$ when $y = 2$ (8 marks)

d) Highlight the general procedure for solving linear first order differential equations of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x. (4 marks)

QUESTION FIVE (20 MARKS)

a) When do we say a complex function $f(z)$ is analytic in some region R? (3 marks)

b) Discuss the Cauchy-Riemann conditions for analyticity of complex functions (3 marks)

c) Determine real and imaginary parts of the complex function $f(z) = z^2 + 1$ (4 marks)

d) For the complex function $f(z) = \frac{1}{(z+5)}$;

   i) State the standard and modified geometric series used to carry out Laurent expansion. (4 marks)

   ii) Determine the Laurent series that are valid in the regions $\{z: |z| < 5\}$ and $\{z: |z| > 5\}$ (6 marks)

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