UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR MAIN EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 204: MATHEMATICAL PHYSICS I

DATE: APRIL 13, 2017 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad \text{for } -\infty < x < \infty,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad \text{for } -\infty < x < \infty,$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad \text{for } -1 < x < 1,$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \quad \text{for } -\infty < x < \infty,$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad \text{for } -1 < x \le 1,$$

$$(1+x)^n = 1 + nx + n(n-1)\frac{x^2}{2!} + n(n-1)(n-2)\frac{x^3}{3!} + \cdots \quad \text{for } -\infty < x < \infty.$$

QUESTION ONE (30 MARKS)

a) Sum the even numbers between 1000 and 2000 inclusive. (5 marks)

b) Consider a ball that drops from a height of 27m and on each bounce retains only a third of its kinetic energy; thus after one bounce it will return to a height of 9m, after two bounces to 3m, and so on. Find the total distance travelled between the first bounce and the M^{th} bounce. (5 marks)

c) Use the difference method to sum the series;

$$\sum_{n=2}^{N} \frac{2n-1}{2n^2(n-1)^2}.$$

(5 marks)

d) Given that the series, $\sum_{n=1}^{\infty} 1/n$ diverges, determine using the **quotient test** whether the following series converges

$$\sum_{n=1}^{\infty} \frac{4n^2-n-3}{n^3+2n}.$$

(5 marks)

e) Solve the equation:

$$\frac{dy}{dx} = \frac{y^2+xy}{x^2}.$$

(5 marks)

f) Evaluate the limit,

$$\lim_{x\to\infty} \left(1-\frac{a^2}{x^2}\right)^{x^2}$$

(5 marks)

QUESTION TWO (20 MARKS)

a) Find the eigenvalues of the matrix,

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

In addition, find the eigenvector corresponding to the eigenvalue $\lambda = 2$. (10 marks)

b) (i) Show that the three vectors (1; 1 + i; 1), (0; i; 1), (1; i; 0) are linearly independent.

(ii) Do they form a basis for $\mathbf{C}^3$? (10 marks)

QUESTION THREE (20 MARKS)

a) Find the solution to the initial value problem of the differential equation:

$$y'' - 3y' + 2y = 3e^{-t},$$

$$y(0) = 1, \; y'(0) = 2.$$

(10 marks)

b) Find the power series solution of the following differential equation about x = 0.

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Suppose two units of mass are placed at (0; 2; 2), one unit of mass at (2; 1; 1), and one unit of mass at (-2; 1; 1).

   i) Find the moment of inertia tensor about the origin. (10 marks)

   ii) Find the principal axes and principal moments of inertia (10 marks)

QUESTION FIVE (20 MARKS)

a) (i) Consider the solid shown (Fig. 1), find the volume below the plane $z = 1+y$, bounded by the coordinate planes and the vertical plane $2x + y = 2$. (5 marks)

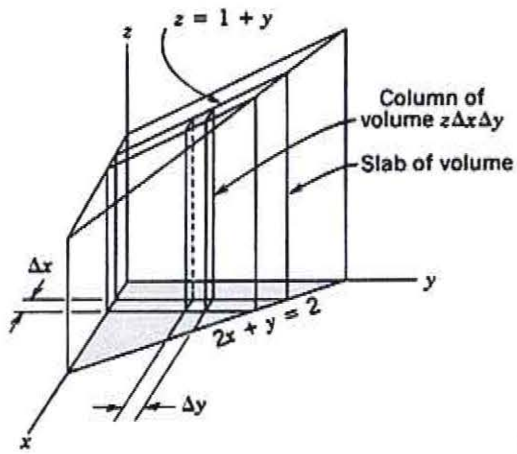

Fig. 1

ii) Find the mass of the solid if the density (mass per unit volume) is $x + z$. (3 marks)

b) Calculate the divergence of the following vector functions:

   i) $\mathbf{v}_a = \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$, (ii) $\mathbf{v}_b = \hat{\mathbf{z}}$,

   ii) $\mathbf{v}_c = z\hat{\mathbf{z}}$. (6 marks)

c) Given the force $\mathbf{F} = xy\,\mathbf{i} - y^2\,\mathbf{j}$, find the work done by $\mathbf{F}$ along the paths indicated in Figure 2 shown below from (0, 0) to (2, 1).

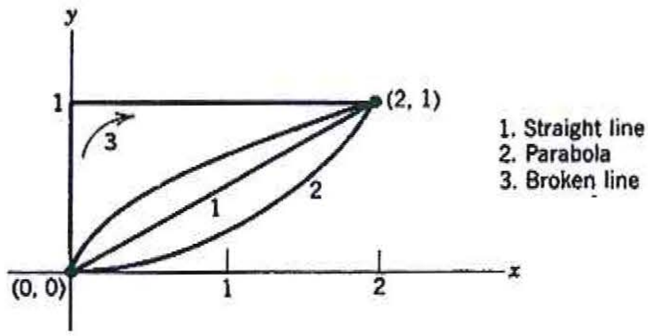

Fig. 2 (6 marks)

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