UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

COMPUTER SCIENCE

CSC 114: DIFFERENTIAL AND INTEGRAL CALCULUS

DATE: DECEMBER 7, 2016 **TIME: 11:00AM-1:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY other TWO Questions

QUESTION ONE (30 MARKS)

a) Simplify $|-4+2i|$. (3 marks)

b) Given $\sin(x-y) = m\sin y$,

change y from an implicit to explicit function.

(4 marks)

c) Find $\frac{dy}{dx}$ of the given function (4 marks)

$$y = x^5 5^x$$

d). The velocity of a moving point changes according to the equation $v = (3t^2 + 2t + 1)m/s$.

Find the length of the path covered by the point during 10 seconds from the start.

(4 marks)

e) Differentiate the given function

$$y = \ln\left|\frac{3x+1}{x+3}\right|$$ (4 marks)

f) Find the length of the arc from $\theta = 0$ *to* $\theta = \frac{\pi}{4}$ of the curve given by

$$x = 3\cos\theta \quad y = 3\sin\theta$$ (4 marks)

g). If $y = \cosh(x^2 - 3x + 1)$, find $\frac{dy}{dx.}$

(3 marks)

h) Prove the De Moivre's theorem, that, if $z_1 = x_1 + iy_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and

$z_2 = x_2 + iy_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Then, $\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\}$ (4 marks)

QUESTION TWO (20 MARKS)

a) Let u and v be differentiable functions of x.

Using the first principle, prove that

i). $\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$ (4 marks)

ii). Given the function

$$y = \sec^{-1}\frac{2x}{3},$$ (4 marks)

Show that $\frac{dy}{dx} = \frac{3}{x\sqrt{4x^2 - 9}}$

b) Differentiate each of the following with respect to x

i). $y = (\tan x)^{\sin x}$ (4 marks)

ii). $y = \frac{3 - 2x}{3 + 2x}$ (4 marks)

c) Evaluate the following indefinite integral

$$\int x^2 e^{2x} dx$$ (4 marks)

QUESTION THREE (20 MARKS)

a) A point moves in the plane according to the law $x = t^2 + 2t,\ \ y = 2t^3 - 6t$

Find $\frac{dy}{dx}$ $when\ t = 0$ (4 marks)

b) Given the function

$$y = \cot^{-1}(\tanh ax)$$

Show that

$$\frac{dy}{dx} = \frac{-a}{\cosh 2ax}$$ (5 marks)

c) Show that

$$\int \tan x dx = \ln|\sec x| + C$$ (6 marks)

d) Find $\int 5\sqrt{5x^2 - 7}\,dx$ (3 marks)

e) Evaluate $\int_1^5 (2x - \frac{1}{x} + 1)dx$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Consider the function $y = x^3 - 6x^2 + 4$.

i) On what intervals is it increasing and decreasing? (2 marks)

ii) Find and classify the relative minimum and maximum points. (6 marks)

iii) Sketch the graph. (2 marks)

b)

i). Find the derivative of $y = \tan^{-1}\sqrt{x}$ with respect to x

(5 marks)

ii). Evaluate $\int \sin 8x \sin 15x dx$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Find the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ *and* $x = 2$ about the $x - axis$ (4 marks)

b) Find the area enclosed by the line $y = 4x$ and the curve $y = x^2$

(6 marks)

c) Find A, B, and C in the partial fraction expansion

$$\frac{x^2 + 1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

and hence evaluate the integral

$$\int \frac{x^2+1}{(x-1)(x-2)(x-3)}\,dx$$ (10 marks)

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