UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 201: MECHANICS II

DATE: NOVEMBER 30, 2016 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Constants: Unless otherwise specified, take;

- Gravitational acceleration, g = 9.8 $m.s^{-2}$
- Speed of light , c = 3.0 x 10^8 $m.s^{-1}$
- Gravitational constant, G = 6.67 X 10^{-11} m^3/s^2. kg.
- Earth's mass, M=5.98 X 10^{24} kg.
- Earth's radius, R_E = 6.37 X 10^6 m.

QUESTION ONE (30 MARKS)

a) Simple harmonic motion is given by $x = A\cos(\omega t - \varphi)$. Give a short but convincing argument showing that the amplitude is given by $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$. (4 marks)

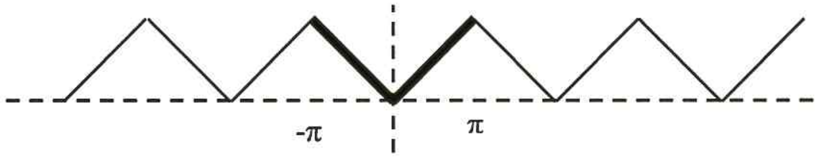
b) The motion of a mass which is attached to a spring and which moves horizontally over a frictionless surface is given by:

$$x = A\cos(\omega t - \phi) \quad \text{With} \quad \omega = \sqrt{\frac{k}{m}}.$$

Give expressions for the kinetic energy and the potential energy of the system, and prove that the total energy has the constant value $E = \frac{1}{2}kA^2$. (4 marks)

c) An oscillation is described in complex notation is $x = (3 - i)e^{i\omega t}$. If this oscillation was expressed in the amplitude-phase form $x = A\cos(\omega t - \phi)$, determine the amplitude and phase constant. (4 marks)

d) Consider a simple pendulum and show that when the bob is displaced a distance x along the arc of its motion, the restoring force along the arc is given by $F \approx -mg\frac{x}{\ell}$. State clearly you approximation is made. By comparing this to Hooke's law, show that the period of a simple pendulum is $2\pi\sqrt{\frac{\ell}{g}}$. (6 marks)



e) Consider the triangle wave shown above defined by the periodic continuation of

$$f(t) = |t| = \begin{cases} -t & \text{if } -\pi < t < 0 \\ t & \text{if } 0 < t < \pi \end{cases}.$$

Calculate the Fourier coefficients a_0, a_k and b_k for this wave pattern, and hence show that:

$$f(t) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \cdots\right).$$ (6 marks)

f) The gravitational force of the earth on an object of mass m is given by $F = -G\frac{mM}{x^2}$. Use this to calculate the velocity of the object as a function of its position if it initially has velocity v_0 at position x_0. Hence determine a formula for the minimum initial speed with which the object must be launched from the earth's surface if it is never to return.

(6 marks)

QUESTION TWO (20 MARKS)

a) Consider a mass attached to a spring and sliding horizontally over a frictionless surface. (i) Show clearly that the acceleration is given by $a = -\omega^2 x$, mentioning any physics laws or definitions you use. (ii) From this equation for the acceleration, find the velocity as a function of position and show this graphically. (7 marks)

b) A mass of 0.2 kg is hung from a spring. (i) If the spring stretches 98 mm due to the weight, determine its spring constant. (ii) If the system then oscillates, what would be the angular frequency if damping was negligible? (iii) Suppose the system has a damping constant only 10% of the value required for critical damping: how long would it take the amplitude envelope to decay to $1/e$ of its original value? (iv) Give an example of an everyday system where critical damping is useful. . (7 marks)

c) An object with mass $m = 1$ kg is attached to a spring with spring constant $k = 100$ N.m^{-1}. The spring is compressed 10 cm and then the object is released with an initial velocity in the direction of the equilibrium position of 1 m.s^{-1}. Assuming there is no friction, find an equation giving the displacement of the object at any time. (6 marks)

QUESTION THREE (20 MARKS)

a) An object with mass 60 g hangs from a spring with natural length 30 cm. The spring stretches 1.5 cm under the weight of the particle. (i) Find the spring constant. (ii) When the particle is set in motion, it experiences a resistive force $R = -\beta v$ and oscillates with damped angular frequency $\omega_0 = 24$ Hz. Calculate the damping constant γ and also the value of β (including units). (5 marks)

b) When a damped oscillator is driven sinusoidally, the steady state motion has amplitude $A = \frac{\omega^2 r}{\sqrt{(\omega^2-\omega_d^2)^2+4\gamma^2\omega_d^2}}$ and lags behind the driving force by a phase constant $\varphi = \tan^{-1}\frac{2\gamma\omega_d}{\omega^2-\omega_d^2}$. Describe what is meant by resonance and derive the resonance angular frequency. Use this to show that the lag angle at resonance is given by $\varphi_R = \tan^{-1}\frac{\sqrt{\omega^2-2\gamma^2}}{\gamma}$. (5 marks)

c) Consider a mass connected to a spring, and which experiences a frictional force proportional to its velocity. By starting with Newton's second law, show that the motion is given by: $x = Ce^{\left(-\gamma+\sqrt{\gamma^2-\omega^2}\right)t} + De^{\left(-\gamma-\sqrt{\gamma^2-\omega^2}\right)t}$. (6 marks)

d) The general equation of motion for a damped oscillator is $a + 2\gamma v + \omega^2 x = 0$. (i)Determine using substitution for which value(s) of r the motion $x = e^{rt}$ would be a solution to this equation. (ii) If for a specific system $a + 10v + 25x = 0$, determine whether the system is undercritically, critically or overcritically damped. (4 marks)

QUESTION FOUR (20 MARKS)

a) Show that the gravitational potential energy U of a ball of mass m at point P along its path, at radial distance R from Earth's centre is given by the expression

$$U = -G\frac{Mm}{R}$$ (7 marks)

b) A mass M is split into two parts, m and M - m, which are then separated by a certain distance. What ratio m/M maximizes the magnitude of the gravitational force between the parts? (5 marks)

c) Derive the formula for the escape speed v of a projectile of mass m, leaving the surface of a planet. (3 marks)

d) The mean diameters of Mars and Earth are 6.9 X 10^3 km and 1.3 X 10^4 km, respectively. The mass of Mars is 0.11 times Earth's mass. (i) What is the value of the gravitational acceleration on Mars? (ii) What is the escape speed on Mars? (5 marks)

QUESTION FIVE (20 MARKS)

a) Suppose that a particle, moving with constant velocity parallel to the x and x' axes sends out two signals as it moves. With the aid of a set of reference axes, use the Lorentz transformation equations to compare the velocities that two observers in different inertial reference frames Sand S' would measure for the same moving particle. Let S' move with velocity v relative to S. Stating the assumptions you made derive the relativistic velocity transformation equation. (10 marks)

b) The origins of two frames S and S' coincide at $t = t' = 0$ and the relative speed is $0.950c$. Two particles collide at coordinates x = 100 km and t = 200 μs according to an observer in frame S. What are the (i) spatial and (ii) temporal coordinate of the collision according to an observer in frame S'? (10 marks)

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