UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN FINANCE

SMA 103: CALCULUS I

DATE: APRIL 4, 2018 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Explain the meaning of limit of a function, hence evaluate,

$$\lim_{x\to 2}\left[\frac{1}{x+2}+\frac{x-2}{x^2-4}\right]$$

(4 marks)

b) Differentiate $y=\dfrac{1}{\sqrt{3x+1}}$ from first principles. (5 marks)

c) Find the equation of the tangent line at the point $(2,3)$ for the curve $x^2+xy-y^2=1$

(4 marks)

d) Consider the parametric equations; $x=\sin\theta$ and $y=\cos\theta$. Find $\dfrac{d^2y}{dx^2}$ (3 marks)

e) If $f(x)=\dfrac{x+5}{3x-7}$. Show that f is bijective. (6 marks)

f) The total cost $c(x)$ associated with production and marketing x units of an item is given by;

$c(x)=0.02x^2+2x+4000$ (Dollars)

i) Compute the marginal cost at $x = 100$ and compare this to the actual cost of producing the 100^{th} unit. (4 marks)

ii) Compute the production level that minimizes the average cost. (4 marks)

QUESTION TWO (20 MARKS)

a) Differentiate each of the following functions with respect to x and simplify your answer.

i) $y = (3x+2)^4$ (2 marks)

ii) $y = \ln \frac{\sqrt{x^2+1}}{\sqrt[3]{x^3+1}}$ (4 marks)

iii) $y = \frac{\cos x}{1-\sin x}$ (3 marks)

iv) $y = \frac{e^x - x^2 + 2}{e^{x+4} + 5}$ (3 marks)

b) A particle is moving along a horizontal line with position function

$s(t) = 2t^3 - 14t^2 + 22t - 5, \quad t \geq 0$

Find the velocity and acceleration and describe the motion of the particle. (6 marks)

QUESTION THREE (20 MARKS)

a) Given $f(x) = 5x,\ g(x) = x + 4$. Find $fg(x),\ gf(x)$ and show that $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$

(8 marks)

b) A box with a square base and an open top is to have volume 62.5cm squared. Neglect the thickness of the material used to make the box and find dimensions that will minimize the amount of material used. (5 marks)

c) Show that If u and v are differentiable function of x then at any point where $v \neq 0$,

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{dy}{dx} - u\frac{dy}{dx}}{v^2}$$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Define continuity of a function $f(x)$ at a point $x = a$ hence or otherwise find the points where the given function is not defined and is therefore discontinuous. $f(x) = \frac{x^2 + x - 2}{x^2 + 2x - 3}$

b) Show for the point a, where discontinuity is removable. (5 marks)

Evaluate $\lim_{x \to 1}\left(\sqrt{1+\frac{1}{x}}-\sqrt{\frac{1}{x}}\right)$ (3 marks)

c) Find the stationary points on the curve $y = x^5 - 5x^4 + 5x^3 - 1$ and determine the nature of each.

(12 marks)

QUESTION FIVE (20 MARKS)

a) State the fundamental theorem of calculus. (3 marks)

b) Evaluate the following integrals

$$\int_1^4 \left(5 - 2t + 3t^2\right)dt$$

(3 marks)

$$\int \frac{8x^5 - 3x}{x^3}dx$$ (3 marks)

$$\int \frac{x}{\sqrt[3]{1+x^2}}dx$$ (3 marks)

c) A rock thrown vertically upward from the surface of the moon at a velocity of 24m/sec (about 86 km/hr) reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

i) Find the rocks velocity and acceleration at time t. (2 marks)

ii) How long does it take to reach its highest point? (2 marks)

iii) How high does the rock go? (2 marks)

iv) How long is the rock aloft? (2 marks)

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