UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS), BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY), BACHELOR OF SCIENCE (INDUSTRIAL CHEMISTRY), BACHELOR OF ECONOMICS, BACHELOR OF EDUCATON (ARTS) AND BACHELOR OF EDUCATION (SCIENCE)

SMA 104/201: CALCULUS II

DATE: APRIL 12, 2018 **TIME: 8:30 AM – 10:30 AM**

INSTRUCTIONS:
Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Evaluate the following:

i) $\int_0^1 x(4-x^2)^3\,dx$. (3 marks)

ii) $\int e^x \sin 2x dx$. (4 marks)

iii) $\int x^2 4^{x^3}\,dx$ (3 marks)

iv) $\int \frac{2x^2}{(x^2-1)(x^2+1)}\,dx$. (5 marks)

b) Find the area bounded by the curves $y = x^2$ and $y = 2x$. (4 marks)

c) Evaluate the area of the surface generated by revolving the curve y = $\sqrt{x}$ between x= 0 and x = 2 about the x-axis. (4 marks)

d) The velocity of a train after leaving a station is as follows:

Time (minutes)	0	2	4	6	8	10	12	14	16
Speed (metres/min.)	0	50	110	160	230	290	360	410	470

Use Simpsons rule to find the distance travelled in the first 16 minutes. (3 marks)

e) Verify the mean value theorem of integral calculus for the function $f(x) = x^2 + 1$ on the interval $[-2,1]$. (4 marks)

QUESTION TWO (20 MARKS)

(a) Determine $f(x)$ given that $f'(x) = 6x^2 - 2x - 1$ and $f(0) = 2$. (3 marks)

(b) Find the volume of the solid of revolution of the area under the curve $y = x^2 + 1$ from $x = 0$ to $x = 2$ about the x-axis. (4 marks)

(c) Determine the area bounded by the curves $y = x^3, y + x = 0$ and $2x - 3y = 20$. (8 marks)

(d) Evaluate $\int \frac{x^2}{(x^2+9)^2} dx$. (5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\int_0^{\pi} \int_0^{\sin x} y dy dx$. (5 marks)

b) Evaluate $\int \sin^3 x \cos^2 x dx$ (4 marks)

c) Determine the length of the arc $y = x^{3/2}$ joining the origin O(1,1) to the point (4,8). (5 marks)

d) Approximate the integral of the function $\int_1^2 \frac{dx}{x}$ using Simpson's rule and obtain the maximum error using 5 strips. (6 marks)

QUESTION FOUR (20 MARKS)

a) Use the trapezoidal rule with four strips to evaluate $\int_1^2 2x^3 dx$. (4 marks)

b) Evaluate $\int \int_R f(x,y) dA$ for $f(x,y) = 1 - 6x^2y, R: 0 \leq x \leq 2, -1 \leq y \leq 1$.(5 marks)

c) Evaluate:

i) $\int x \ln x dx$. (4 marks)

ii) $\int \frac{x-1}{x^2-3x} dx$. (4 marks)

iii) $\int \frac{\sec^2 x}{\tan^5 x} dx$. (3 marks)

QUESTION FIVE (20 MARKS)

a) Find the volume of revolution bounded by the region $y = \sqrt{x}$ the line $x = 1$ and $x = 4$ about the line $y = 1$. (5 marks)

b) Evaluate:

i. $\int \frac{1}{\sqrt{4x-x^2}} dx$ (5 marks)

ii. $\int \frac{4x^3+x^2+x+1}{4x^4+5x^2+1} dx$ (5 marks)

c) Use Maclaurin's theorem to expand $f(x) = \ln(1 + x)$ in ascending powers of x up to the term in x^5. (5 marks)

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SMA 104

DEFINITIONS AND FORMULAE

Area and volume formulae

Volume of a cone or pyramid $= \frac{1}{3}Ah$, where A = base area, h = height of vertex.
Area of curved surface of a cone $= \pi rl$, where l = slant height, r = base radius.
Volume of a sphere $= \frac{4}{3}\pi r^3$.
Surface area of a sphere $= 4\pi r^2$.
Area of a spherical zone (between planes distance h apart) $= 2\pi rh$.

Trigonometry

$$\sec\theta = \frac{1}{\cos\theta}; \quad \csc\theta = \frac{1}{\sin\theta}; \quad \tan\theta = \frac{\sin\theta}{\cos\theta}; \quad \cot\theta = \frac{1}{\tan\theta}.$$

$$\cos^2\theta + \sin^2\theta = 1; \quad 1+\tan^2\theta = \sec^2\theta; \quad \cot^2\theta + 1 = \csc^2\theta.$$

$$\sin(\theta\pm\phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi; \quad \cos(\theta\pm\phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi;$$

$$\tan(\theta\pm\phi) = \frac{\tan\theta \pm \tan\phi}{1\mp\tan\theta\tan\phi} \quad [\theta\pm\phi \neq (k+\tfrac{1}{2})\pi].$$

$$\sin 2\theta = 2\sin\theta\cos\theta; \quad \cos 2\theta = \cos^2\theta - \sin^2\theta; \quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \quad [\theta \neq (\tfrac{1}{2}k+\tfrac{1}{4})\pi].$$

$$2\cos^2\theta = 1+\cos 2\theta; \quad 2\sin^2\theta = 1-\cos 2\theta.$$

If $t = \tan\frac{1}{2}\theta$, then $\sin\theta = \frac{2t}{1+t^2}$; $\cos\theta = \frac{1-t^2}{1+t^2}$; $\tan\theta = \frac{2t}{1-t^2}$; $\frac{d\theta}{dt} = \frac{2}{1+t^2}$.

$$2\sin\theta\cos\phi = \sin(\theta+\phi)+\sin(\theta-\phi);$$

$$2\cos\theta\cos\phi = \cos(\theta+\phi)+\cos(\theta-\phi);$$

$$2\sin\theta\sin\phi = \cos(\theta-\phi)-\cos(\theta+\phi).$$

$$\sin\alpha+\sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta); \quad \sin\alpha-\sin\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\alpha-\beta);$$

$$\cos\alpha+\cos\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta); \quad \cos\alpha-\cos\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\beta-\alpha).$$

In the triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R;$$

$$a^2 = b^2+c^2-2bc\cos A, \text{ etc.};$$

$$\tan\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}, \text{ etc.}; \quad \text{area} = \sqrt{\{s(s-a)(s-b)(s-c)\}};$$

where $s = \frac{1}{2}(a+b+c)$.

Ranges of the inverse functions:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1}x \leqslant \tfrac{1}{2}\pi; \quad 0 \leqslant \cos^{-1}x \leqslant \pi; \quad -\tfrac{1}{2}\pi < \tan^{-1}x < \tfrac{1}{2}\pi.$$

Indefinite integrals of common functions

[In the following we take $a > 0$ and omit the additive constant.]

$f(x)$	$\int f(x)dx$	
$x^n \ (n \neq -1)$	$x^{n+1}/(n+1)$	
$1/x$	$\ln x$ if $x > 0$, $\ln(-x)$ if $x < 0$ (i.e. $\ln\lvert x\rvert$, $x \neq 0$)	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	
$\dfrac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\dfrac{x}{a}$	For logarithmic forms of inverse hyperbolic functions see p. 3.
$\dfrac{1}{\sqrt{(x^2-a^2)}}$	$\cosh^{-1}\dfrac{x}{a}$ if $x > a$, $-\cosh^{-1}\left(\dfrac{-x}{a}\right)$ if $x < -a$	For logarithmic forms of inverse hyperbolic functions see p. 3.
$\dfrac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\dfrac{x}{a}$	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\tan x$	$\ln\lvert\sec x\rvert$	
$\cot x$	$\ln\lvert\sin x\rvert$	
$\sec x$	$\ln\lvert\sec x+\tan x\rvert = \ln\lvert\tan(\frac{1}{2}x+\frac{1}{4}\pi)\rvert$	
$\csc x$	$\ln\lvert\tan\frac{1}{2}x\rvert$	
$e^{ax}\sin bx$	$\dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$	
$e^{ax}\cos bx$	$\dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$	
$\sin^2 x$	$\frac{1}{2}(x-\frac{1}{2}\sin 2x)$	
$\cos^2 x$	$\frac{1}{2}(x+\frac{1}{2}\sin 2x)$	
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	

Integration by parts

$$\int u\frac{dv}{dx}\,dx = uv - \int\frac{du}{dx}\,v\,dx.$$

Reduction formulae for trigonometric integrals

$$\int_0^{\frac{1}{2}\pi}\sin^m x\,dx = \frac{m-1}{m}\int_0^{\frac{1}{2}\pi}\sin^{m-2}x\,dx; \quad \int_0^{\frac{1}{2}\pi}\cos^m x\,dx = \frac{m-1}{m}\int_0^{\frac{1}{2}\pi}\cos^{m-2}x\,dx;$$

$$\int_0^{\frac{1}{2}\pi}\sin^m x\cos^n x\,dx = \frac{m-1}{m+n}\int_0^{\frac{1}{2}\pi}\sin^{m-2}x\cos^n x\,dx = \frac{n-1}{m+n}\int_0^{\frac{1}{2}\pi}\sin^m x\cos^{n-2}x\,dx.$$

[These results hold provided that the exponents in the reduced form are greater than -1. There are analogous reduction formulae with other intervals of integration $(\frac{1}{2}k_1\pi, \frac{1}{2}k_2\pi)$ with k_1, k_2 integral.]