UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

SMA 108: DISCRETE MATHEMATICS

DATE: APRIL 10, 2018 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other TWO Questions

QUESTION ONE (30 MARKS)

a) Simplify $\frac{D_6}{6!}$ (3 marks)

b) Mr Jamleck has 12 children but his car can only hold five people including the driver. In how many ways can he select four of his children to accompany him for shopping

(2 marks)

c) Find the gcd(91, 287) using the Euclidean Algorithm (3 marks)

d) Find an integer $0 \leq a < 18$ such that $3958 \equiv a(mod 18)$ (3 marks)

e) Let A be the set $\{1,2,3,4\}$. Which ordered pairs are in the relation

i) $R = \{(a, b)|a \text{ divides } b\}$ (2 marks)

ii) $R = \{(a, b)|a + b \leq 3\}$ (1 mark)

iii) $R = \{(a, b)|a = b + 1\}$ (1 mark)

f) Proof by cases that if n is an odd integer then there is an integer *m* such that

$n = 4m + 1 \text{ or } n = 4m + 3$ (4 marks)

g) Let a function *f* be defined on the set of real numbers as $f(x) = \frac{-6x-7}{10-2x}$

find $f^{-1}(-\frac{1}{2})$ (4 marks)

h) Write in words the inverse, contrapositive and converse of the conditional statements "if he works hard, then he will pass the examination" (3 marks)

i) A club has ten members. In how many ways can they choose four officers consisting of a president, vice - president, secretary and treasurer? (2 marks)

j) The chairs of a hall are to be labelled with a letter and a positive integer n such that $1 \leq n \leq 100$.What is the largest number of chairs that can be labelled differently? (2 marks)

QUESTION TWO (20 MARKS)

a)

i) Is the statement $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? (7 marks)

ii) What conclusion can you make about the compound statement?

$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \rightarrow q) \rightarrow r]$ (3 marks)

b) Use mathematical induction to show that $1 + x + x^2 + \cdots + x^n = \frac{x^{n+1}-1}{x-1}$ for any non-negative integer n and any real number $x \pm 1$ (6 marks)

c) Let R be the relation from $\{1,2,3\} to \{1,2,3,4\}$ with $R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$ and S be the relation from $\{1,2,3,4\} to \{0,1,2,\}$ with $S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$ find

i) $S \circ R$ (2 marks)

ii) R^2 (2 marks)

QUESTION THREE (20 MARKS)

a) An urn contains fifteen red numbered balls and ten white numbered balls. A sample of five balls is selected

i) How many different samples are possible? (2 marks)

ii) How many samples contain three red balls and two white balls? (3 marks)

iii) How many samples contain all red balls? (2 marks)

b) Consider the following relations on $\{1,2,3,4\}$

$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$

$R_2 = \{(1,1), (1,2), (2,1)\}$

$R_3 = \{(1,1), (1,2), (2,1), (1,4), (2,2), (3,3), (4,1), (4,4)\}$

$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,4)\}$

$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$. Which of these relations are

i) Reflexive (1 mark)

ii) Symmetric (1 mark)

iii) Antisymmetric (1 mark)

iv) Transitive (1 mark)

c) Determine whether the sequence a_n is a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if $a_n = 4^n$ hence find a_5 (5 marks)

d) Which rule of inference is used in each of the following arguments?

i) If it snows today, the university will close. The university is not closed today. Therefore it did not snow today (2 marks)

ii) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore if I go swimming then I will sunburn (2 marks)

QUESTION FOUR (20 MARKS)

a) Let $f: R \to R$ be defined as $f(x) = -6x^2 + 5$.

Determine whether f is one to one or onto (5 marks)

b) Let $R_1 = \{(1,2), (2,3), (3,4)\}$ and

$R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4)\}$ be relations from $\{1,2,3\}$ to $\{1,2,3,4\}$ find

i) $R_2 - R_1$ (1 mark)

ii) $R_1 \cup R_2$ (1 mark)

iii) $R_1 \cap R_2$ (1 mark)

c) Proof by contradiction that $\sqrt{3}$ is not a rational number (6 marks)

d) Use the truth table to determine whether the following argument is valid

If there is gas in the car, then I will go to the store

If I go to the store, then I will get a soda

There is gas in the car

I will get a soda (6 marks)

QUESTION FIVE (20 MARKS)

a) In a freshman class of 586 students, 226 take Calculus, 201 take Discrete Mathematics and 206 take Algebra. 106 students take Calculus and Discrete Mathematics, 121 take Discrete Mathematics and Algebra, 96 take Calculus and Algebra while 76 take all the three courses. Using the principle of Inclusion and Exclusion find,

i) How many freshmen take at least one of the courses (4 marks)

ii) How many freshmen take none of the three courses (2 marks)

iii) How many freshmen take calculus and discrete mathematics but not Algebra (2 marks)

b) Using generating functions, solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2},\ n \geq 2$ given $a_0 = 3, a_1 = -2$ (8 marks)

c) Let $A = \{2,3\}$ and $B = \{1,2,3\}$ define a binary relation R from A to B as follows $R = \{(a, b) \in A \times B | a > b\}$ find

i) The ordered pair in R (2 marks)

ii) The domain and range of R (2 marks)

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