UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

SMA 204: LINEAR ALGEBRA II.

DATE: APRIL 10, 2018 **TIME: 8:30 AM – 10:30 AM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Briefly define the following terms;

i) A bilinear form of a vector space V. (2 marks)

ii) A linear functional of a vector space V. (2 marks)

b) Find the determinant of a 2×2 square matrix $A=\begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}$ by cofactor expansion along the first row. (4 marks)

c) Prove that if two $n\times n$ square matrices A and B are similar, then they have the same characteristic polynomial. (4 marks)

d) Find the minimal polynomial of a 2×2 square matrix $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. (4 marks)

e) Find the quadratic form $q(x)$ that corresponds to the 2×2 square symmetric matrix

$A=\begin{pmatrix} 5 & -3 \\ -3 & 8 \end{pmatrix}$. (3 marks)

f) Show that if A is square matrix and λ is an eigenvalue of A, then $E_A(\lambda) = Ker(\lambda I - A)$. (4 marks)

g) Show that if a matrix A is orthogonal, then $\det(A) = \pm 1$. (4 marks)

h) Let $T : \Re^n \to \Re$ be defined by $Tx = \sum_{i=1}^{n} x_i$ where $x = (x_1, x_2, ..., x_n)$. Show that T is a linear functional on $V = \Re^n$. (3 marks)

QUESTION TWO (20 MARKS)

a) Given a 3×3 square matrix $\begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. Find;

i) The characteristic polynomial of A. (3 marks)

ii) Eigenvalues of A. (3 marks)

iii) Eigenvectors of A. (5 marks)

b) Find the dual basis $\beta^* = \{\phi_1, \phi_2\}$ given the basis $\beta = \left\{ \overline{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \overline{v_2} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$ of $\Re^2$. (6 marks)

c) Verify that $Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is an orthogonal matrix. (3 marks)

QUESTION THREE (20 MARKS)

a) Find the determinant of a 5×5 square matrix $M = \begin{pmatrix} 2 & 3 & 4 & 7 & 8 \\ -1 & 5 & 3 & 2 & 1 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & 3 & -1 & 4 \\ 0 & 0 & 5 & 2 & 6 \end{pmatrix}$ by use of an appropriate method. (7 marks)

b) Prove that a bilinear form f is alternating if and only if it is skew symmetric. (3 marks)

c) Given a 5×5 square block diagonal matrix $A=\begin{pmatrix}2&5&0&0&0\\0&2&0&0&0\\0&0&4&2&0\\0&0&3&5&0\\0&0&0&0&7\end{pmatrix}$. Find;

i) The characteristic polynomial of A. (4 marks)

ii) The minimal polynomial of A. (6 marks)

QUESTION FOUR (20 MARKS)

a) Let A and B be 3×3 square matrices given by $A=\begin{pmatrix}2&1&0\\0&2&1\\0&0&2\end{pmatrix}$ and $B\begin{pmatrix}2&1&0\\0&2&0\\0&0&2\end{pmatrix}$.

Determine whether A and B are similar. (6 marks)

b) Let $P_1=(a+bt : ab\in\Re\}$ be the vector space of real polynomials of degree ≤ 1. Find the basis $\{\overline{v_1},\overline{v_2}\}$ of V that is dual to the basis $\{\phi_1,\phi_2\}$ of V^* defined by $\phi_1(f(t))=\int_0^1 f(t)dt$ and $\phi_2(f(t))=\int_0^2 f(t)dt$. (4 marks)

c) Given a 2×2 square matrix $A=\begin{pmatrix}5&-6\\3&4\end{pmatrix}$,

i) Show that A is diagonizable. (6 marks)

ii) Compute A^{10} using results in $c(i)$. (4 marks)

QUESTION FIVE (20 MARKS)

a) Find the orthonormal basis set by use of the Gram Schmidt orthogonalization process given that $\overline{v_1}=\begin{pmatrix}1\\2\\-1\end{pmatrix},\overline{v_2}=\begin{pmatrix}0\\1\\-1\end{pmatrix}$ and $v_3=\begin{pmatrix}3\\-7\\1\end{pmatrix}$ are basis of an inner product space V. (6 marks)

b) Show that the characteristic polynomial $\chi_A(t)$ and the minimal polynomial $m_A(t)$ of A have the same irreducible factors. (4 marks)

c) Find the polynomial matrix of a 3×3 square matrix $A=\begin{pmatrix}-1 & 3 & 2\\ 1 & 0 & -2\\ -3 & 1 & 1\end{pmatrix}$ given that

$P(x)=14+19x-3x^2-7x^3+x^4$. (7 marks)

d) Express a function $f(\bar{u},\bar{v})=3x_1y_1-2x_1y_3+5x_2y_1+7x_2y_2-8x_2y_3+4x_2y_3-6x_3y_3$ in matrix notation, given that $\bar{u}=(x_1,x_2,x_3)$ and $\bar{v}=(y_1,y_2,y_3)$. (3 marks)

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