UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF EDUCATION (ARTS), BACHELOR OF SCIENCE (STATISTICS), BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY) AND BACHELOR SCIENCE (INDUSTRIAL CHEMISTRY)

SMA 208: ORDINARY DIFFERENTIAL EQUATIONS 1

DATE: APRIL 6, 2018 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Given the differential equation

$$\frac{d^2y}{dx} + 2x\frac{dy}{dx} + \sqrt[3]{xy} = 0 \qquad \text{(3 marks)}$$

determine the degree and linearity.

b) By use of examples illustrate the difference between a **first order first degree** differential equation and **first order linear** differential equation. (3 marks)

c) Obtain a differential equation associated with the primitive below

$$y = C_1e^x\cos x + C_2e^x\sin x \qquad \text{(5 marks)}$$

d) Given the differential equation

$$\frac{2x}{y^3}dx + \frac{(y^2 - 3x^2)}{y^4}dy = 0 \qquad \text{(4 marks)}$$

Show that the differential equation is exact and that the solution is

$$x^2 - y^2 = cy^3$$

e) Find the solution to the given first order first degree differential equation.

$$\frac{dy}{dx} = 3x - 2y + 4$$ (5 marks)

f) If you are using method of undetermined coefficients to solve this equation, what is your best guess for the form of the particular solution?

$$y'' - 10y' + 25y = e^{5t}$$ (3 marks)

g) Find the general solution of the differential equation

$$10y'' + 7y' - 12y = 0$$ (3 marks)

h) A cup of tea at $150^{\circ}F$ is left in a room temperature of $80^{\circ}F$. At time $t = 0$, the tea is cooling at $14^{\circ}F$ per minute. Assuming that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings, find the function that models the cooling of the tea. (4 marks)

QUESTION TWO (20 MARKS)

a) Given the differential equation, test for exactness and hence solve it.

$$\frac{dy}{dx} = \frac{2y^2 - 3xy}{x^2 - 2xy}$$ (8 marks)

b) Solve the differential equation

$$dx + (1 - x^2)\cot y\, dy = 0$$ (6 marks)

c) If the population of a country doubles in 50 years, in how many years will it treble under the assumption that the rate of increase is proportional to the number of inhabitants.

(6 marks)

QUESTION THREE (20 MARKS)

a) Find the solution of the system of equations

$$\frac{dx}{dt} + \omega y = 0$$

$$\frac{dy}{dt} - \omega x = 0$$ (6 marks)

b) Show that $\cos y$ is an integrating factor of the non-exact differential equation

$$\frac{dy}{dx}=\frac{e^{x}}{e^{x}\tan y+3y\sec y}$$ (7 marks)

c) Transform the Bernoulli equation

$$\frac{dy}{dx}+P(x)y=Q(x)y^{n}$$

into first degree, first order and hence solve the equation

$$x\frac{dy}{dx}+y=y^{2}x^{2}$$

(7 marks)

QUESTION FOUR (20 MARKS)

a) Find the general solution of the following differential equation

$$\frac{dy}{dx}=\frac{-2x-y+6}{-x-2y+3}$$ (8 marks)

b) Solve the differential equation

$$xyp^{2}+(x^{2}+xy+y^{2})p+x^{2}+xy=0$$

where $p=\frac{dy}{dx}$ (5 marks)

c) Find a power series solution to the differential equation

$$y''+y=0$$

(7 marks)

QUESTION FIVE (20 MARKS)

a) Use the method of undetermined coefficients to find the solutions of the differential equation.

$$\frac{d^{2}y}{dx^{2}}+4y=3\cos 2x$$ (10 marks)

b) By the use of the method of variation of parameters, obtain the general solution of the equation

$$\frac{d^{2}y}{dx^{2}}+4y=\cot 2x$$ (10 marks)

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