



UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF ECONOMICS

SMA 232: INTRODUCTION TO NUMERICAL METHODS.

DATE: APRIL 5, 2018

TIME: 11:00 AM – 1:00 PM

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

- a) Briefly define the following terms.
- i) Inherent error. (2 marks)
 - ii) Round off error. (2 marks)
- b) Approximate the relative error in $z = x - y$ when stored in 4-digit mantissa, given that;
 $x = 0.1234 \times 10^4$ and $y = 0.1232 \times 10^4$ (4 marks)
- c) Use the Newton Raphson formula to obtain an estimate of a root of the function
 $f(x) = x + 2 + \ln x$. (3 marks)
- d) Use the trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. (3 marks)
- e) Use Gaussian elimination to solve the following system of linear equations
- $$\begin{cases} y + 3z = 9 \\ 2x + 2y - z = 8 \\ -x + 5z = 8 \end{cases} \quad (4 \text{ marks})$$

f) Solve $y' = y - x^2$, $y(0) = 1$ by Picard's method up to the third approximation. Hence, find the value of $y(0.1)$. (4 marks)

g) Use Newton's forward interpolation formula to find a cubic polynomial in x which takes values $-3, 3, 11, 27, 57$ and 107 when $x = 0, 1, 2, 3, 4$ and 5 respectively. (4 marks)

h) Use backward differences formula to find $f'(2.2)$ from the following tabular data. (4 marks)

x	1.4	1.6	1.8	2.0	2.2
y	4.0552	4.9530	6.0496	7.3981	9.0250

QUESTION TWO (20 MARKS)

a) Solve the following system of linear equations by LU decomposition $\begin{cases} 2x + 3y + z = 9 \\ x + 2y + 3z = 6 \\ 3x + y + 3z = 8. \end{cases}$ (10 marks)

b) Use Gauss' forward difference formula to find the value of $e^{1.17}$ from the following tabular data. (5 marks)

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

c) Use Euler method to solve the equation $y' = 2xy + 1$ with $y(0) = 0, h = 0.02$ for $x = 0.1$ (5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\int_0^6 \frac{1}{3+x^2} dx$ using Simpson's three eight rule. (7 marks)

b) Find a real root of the equation $x = e^{-x}$ using the Newton - Raphson method. (5 marks)

c) Using modified Euler's method, determine the value of y when $x = 0.1$ given that $y' = x^2 + y; y(0) = 1$. (Take $h=0.05$) (8 marks)

QUESTION FOUR (20 MARKS)

- a) Show that any value of a function f can be expressed in terms of f_n and its backward differences. (5 marks)
- b) Find a real root of the equation $x - e^{-x} = 0$ using secant method. (8 marks)
- c) Use second order Runge Kutta method with $h = 0.1$ to find $y = (0.2)$ given $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$. (7 marks)

QUESTION FIVE (20 MARKS)

- a) Use Taylor's series method to find $y(0.1), y(0.2), z(0.1), z(0.2)$, given that $\frac{dy}{dx} = x + z, \frac{dz}{dx} = x - y^2, y(0) = 2, z(0) = 1$ and assuming a height of 0.1. (8 marks)
- b) Use Gauss Siedel iteration to solve the following system of equations in three steps from 1,1,1.

$$\begin{cases} 10x + y + z = 6 \\ x + 10y + z = 6 \\ x + y + 10z = 6. \end{cases} \quad (8 \text{ marks})$$

- c) From the following table of values x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(4 marks)

--END--

