UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ECONOMICS

SMA 272: INTRODUCTION TO OPTIMIZATION

DATE: APRIL 12, 2018 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Define the following solution terminologies:

i) Feasible solution (2 marks)

ii) Feasible region (2 marks)

iii) Infeasible solution (2 marks)

iv) Optimal solution (2 marks)

v) A corner-point feasible solution (2 marks)

b) Slim plc produce three kinds of malted drink powder. One of these they sell as a health drink because it has less sugar; one they sell to hospitals as an invalid food as it has added vitamins; the third one is a standard product. The main ingredients, with their costs and normal weekly availabilities, are given in the table below.

	Required kg per kg of product			*Sale price*
	Sugar	*Malt*	*Skimmed milk*	*per Kg*
Standard drink	0.30	0.30	0.35	1.00
Health drink	0.15	0.25	0.55	1.20
Invalid drink	0.15	0.30	0.25	1.50
Costs per kg of raw material(One penny = 0.01 shillings)	20p	60p	50p	
Availability per week of raw materials, kg.	1000	1250	2200	

Other variable costs are 10p per kg for the standard drink, 9p per kg for the health drink and 12p per kg for the invalid drink.

Required:

Formulate the linear programming model for this problem with the objective of maximizing total contribution per week (5 marks)

c) A mathematical problem is formulated as follows:

Maximize $Z = 3x_1 + 5x_2$

Subject to the restrictions

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

And

$$x_1 \geq 0,\ x_2 \geq 0$$

i) Demonstrate that the feasible region is bounded (5 marks)

ii) Use the graphical method to solve for the optimal solution (5 marks)

d) Find the stationary value(s) of the following function:

$y = -x^4$ (5 marks)

QUESTION TWO (20 MARKS)

For each of the following multivariable functions (1) find the critical points, and (2) determine if at these points the functions is maximized, is minimized, is at an inflection point, or is at a saddle point.

a) $f(x,y) = 60x + 34y - 4xy - 6x^2 - 3y^2 + 5$

   i) Obtain the critical point(s) (5 marks)

   ii) Evaluate the critical point(s) for a maximum, minimum, inflection point or a saddle point (5 marks)

b) $z(x,y) = 3x^3 - 5y^2 - 225x + 70y + 23$

   i) Obtain the critical point(s) (5 marks)

   ii) Evaluate the critical point(s) for a maximum, minimum, inflection point or a saddle point (5 marks)

QUESTION THREE (20 MARKS)

An organization has three machine shops viz. A, B and C and it produces three product viz. X, Y and Z using these three machine shops. Each product involves the operation of the machine shops. The time available at the machine shops A, B and C are 100, 72 and 80 hours respectively. The profit per unit of product X, Y and Z is $22, $6 and $2 respectively. The following table shows the time required for each operation for unit amount of each product.

Products \ Machine	A	B	C
X	10	7	2
Y	2	3	4
Z	1	2	1

Required:-

a) Formulate the above linear programming problem (5 marks)

b) Use the Simplex method to determine an appropriate product mix so as to maximize the profit. (10 marks)

c) Interpret the shadow prices obtained above (5 marks)

QUESTION FOUR (20 MARKS)

a) A constrained optimization problem is given as follows:

$f(x, y, z) = 4xyz^2$

Subject to

$x + y + z = 56$

i) Use the Lagrange multiplier to optimize the above functions subject to the given constraint (5 marks)

ii) Estimate the effect on the value of the objective function from a 1-unit change in the constant of the constraint (5 marks)

b)

i) What combination of goods x and y should a firm produce to minimize costs when the joint cost functions is $c = 6x^2 + 10y^2 - xy + 30$ and the firm has a production quota of $x + y = 34$? (5 marks)

ii) Estimate the effect on costs if the production quota is reduced by 1 unit (5 marks)

QUESTION FIVE (20 MARKS)

a) Given the following total revenue and total cost functions for different firms: (1) Set up the profit function, (2) find the critical values(s) where π is at a relative extremum and test the second-order condition, and (3) calculate the maximum profit.

i) $TR = 1400Q - 6Q^2$ $TC = 1500 + 80Q$ (5 marks)

ii) $TR = 4350Q - 13Q^2$ $TC = Q^3 - 5.5Q^2 + 150Q + 675$ (5 marks)

b) Find (1) the relative extrema for the following functions and (2) determine if at the critical value(s) the function is a relative maximum or minimum

i) $f(x) = -7x^2 + 126x - 23$ (5 marks)

ii) $f(x) = 3x^3 - 36x^2 + 135x - 13$ (5 marks)

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