# UNIVERSITY OF EMBU

**2017/2018 ACADEMIC YEAR**

**SECOND SEMESTER EXAMINATIONS**

**FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE**

**SMA 402: TOPOLOGY II**

**DATE: APRIL 3, 2018** **TIME: 11:00 AM – 1:00 PM**

**INSTRUCTIONS:**

**Answer Question ONE and ANY Other TWO Questions.**

**QUESTION ONE (30 MARKS)**

a) Let X be a nonempty set. Explain what is meant by a class of subsets of X say $\mathcal{A} = \{G_i\}$ is an open cover of A where A$\subset X$. (2 marks)

b) Let A be any finite subset of a topological space (X, $\tau$). Show that A is compact. (3 marks)

c) Explain what is meant by a class $\{A_i\}$ of sets to have a finite intersection property and hence show if or not the class $\mathcal{A} = \{(0,1), (0,\frac{1}{2}), (0,\frac{1}{3}), \ldots\}$ of intervals in $\mathbb{R}$ has a finite intersection property. (4 marks)

d) Show that the open interval A = (0,1) on the real line $\mathbb{R}$ with the usual topology is not sequentially compact. (3 marks)

e) Let A be a compact subset of a Haursdorff space and suppose p$\in X\backslash A$, show that there exists open sets G and H such that p$\in G$, A$\subset H$ and G$\cap H = \emptyset$. (4 marks)

f) State without proof the Urysohn's lemma. (1 mark)

g) Define the following terms.

  i) Closed path (1 mark)

  ii) Totally disconnected space (1 mark)

h) Show that if A and B are non-empty separated sets, then A$\cup B$ is disconnected. (3 marks)

i) Prove that a regular space need not to be $T_1$-space. (4 marks)

j) Let $f : I \rightarrow X$ be a path from a to b and let $g : I \rightarrow X$ be a path from b to c. Show that the juxtaposition of the two paths f and g denoted by $f * g$ is the function $f * g : I \rightarrow X$ defined by

$$f * g = \begin{cases} f(2s) & 0 \leq s \leq \frac{1}{2} \\ g(2s-1) & \frac{1}{2} \leq s < 1 \end{cases}$$

is a path from a to c. (4 marks)

## QUESTION TWO (20 MAR KS)

a) Prove that the following statements are equivalent

   i) X is compact

   ii) For every class $\{f_i\}$ of closed subsets of X, $\cap f_i = \emptyset$ implies $\{f_i\}$ contains a finite subclass $\{f_{i1}, f_{i2}, ... f_{im}\}$ with $f_{i1} \cap f_{i2} \cap ... \cap f_{im} = \emptyset$. (8 marks)

b) Show that every compact subset A of Hausdorff space is closed. (3 marks)

c) Let X be a Hausdorff space. Prove that every convergent sequence in X has a unique limit. (4 marks)

d) Explain what is meant by a subset of a topological space X is compact and hence show that if A is any finite subset of a topological space X, then A is sequentially compact. (5 marks)

## QUESTION THREE (20 MARKS)

a) Differentiate between the terms separated sets and a disconnected set. (4 marks)

b) Consider the real line $\mathbb{R}$ with the usual topology. Show that $\mathbb{R}$ is locally compact. (4 marks)

c) Prove that every arcwise connected set A is connected. (5 marks)

d) Consider the topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ on X = {a,b,c} and the topology $\tau^* = \{Y, \emptyset, \{0\}\}$ on Y = {u,v}

   i) Determine the defining subbase of product topology XxY. (5 marks)

   ii) Determine the defining base for the product topology on XxY. (2 marks)

## QUESTION FOUR (20 MARKS)

a) Prove that the component of a totally disconnected space X are singleton subsets of X. (4 marks)

b) Let $G \cup H$ be a disconnection of A. Show that $A \cap G$ *and* $A \cap H$ are separated sets (5 marks)

c) Let X be sequentially compact. Show that X is also countably compact. (6 marks)

d) Let A be a subset of a topological space $(X,\tau)$ and let $\tau_A$ be the relative topology on A, then show that A is connected with respect to $\tau$ if and only if A is connected with respect to $\tau_A$. (5 marks)

**QUESTION FIVE (20 MARKS)**

a) Let $\mathcal{G}$ be a base for a second countable space X. Prove that $\mathcal{G}$ is reducible to a countable base for X. (6 marks)

b) Show that any infinite subset A of a discrete topological space X is not compact. (6 marks)

c) Define a product topology. (2 marks)

d) Show that a function $F: X \rightarrow Y$ from a topological space $X = \prod x_i$ is continuous if and only if for every projection $\pi_i: X \rightarrow X_i$, the composition $\pi_i \circ F: Y \rightarrow X_i$ is continuous. (6 marks)

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