UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 539: NUMERICAL ANALYSIS

DATE: APRIL 11, 2018 **TIME: 2:00 PM – 5:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions. Exam Duration is 3 Hours.

QUESTION ONE (30 MARKS)

a) Consider the function $f(x) = \frac{10}{1-x^2}$ for all $x \in \mathbb{R}$.

i) Find the condition number of f. (3 marks)

ii) Is the function ill or well-conditioned? (2 marks)

b) Consider the vector $x = (1, 0, -1, 2)$. Find

i) $\|x\|_1$. (1 mark)

ii) $\|x\|_2$. (1 mark)

iii) $\|x\|_\infty$. (1 mark)

c) Compute the Frobenius norm of $A = \begin{bmatrix} 5 & 9 \\ -2 & 1 \end{bmatrix}$. (3 marks)

d) Find the eigenvalues of $A(t) = \begin{pmatrix} -1 + \frac{3}{2}\cos^2(t) & 1 - \frac{3}{2}\cos(t)\sin(t) \\ -1 - \frac{3}{2}\cos(t)\sin(t) & -1 + \frac{3}{2}\sin^2(t) \end{pmatrix}$.

(4 marks)

e) Consider evaluating a function $f(x)$ at the approximate value x_A rather than at x. How well does $f(x_A)$ approximate $f(x)$? (4 marks)

f) Use Gaussian elimination to solve the following system of linear equations. (6 marks)

$$4x_1 - 2x_2 + x_3 = 15,$$

$$-3x_1 - x_2 + 4x_3 = 8,$$

$$x_1 - x_2 + 3x_3 = 13.$$

g) Describe what is meant by instability or ill-conditioning of systems. (5 marks)

QUESTION TWO (20 MARKS)

Let $x_T = x_A + \varepsilon$ and $y_T = y_A + \eta$ be some positive numbers where (x_T, y_T) denote true values of (x, y); (x_A, y_A) denote approximate values of (x, y); and (ε, η) denotes the deviations of (x_T, y_T) and (x_A, y_A). Find the relative error, E_r, with respect to the following;

a) $x_A + y_A$. (3 marks)

b) $x_A - y_A$. (3 marks)

c) $x_A \times y_A$ (Give your answer in terms of $E_r(x_A)$ and/or $E_r(y_A)$). (5 marks)

d) x_A / y_A (Give your answer in terms of $E_r(x_A)$ and/or $E_r(y_A)$). (5 marks)

e) Comment on the results obtained in (a)-(d) above. (4 marks)

In the above, $E_r(\,)$ denotes the relative error with respect to the identity under brackets.

QUESTION THREE (20 MARKS)

a) Let x_A and y_A denote approximate values of x and y, respectively, and let x_T and y_T denote true values of x and y, respectively. Determine how the relative error propagates with division. (5 marks)

b) Consider the following system of equations:

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = -0.03,$$

$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = 0.12,$$

$$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = -0.10.$$

i) Represent the system in the form, $A\vec{x} = \vec{b}$, where A is a 3×3 matrix, $\vec{x} = [x_1, x_2, x_3]^T$, and $\vec{b}$ is a 3×1 vector of constants. (2 marks)

ii) Use Gaussian elimination method to find the inverse of the matrix A formed in (a) above. (10 marks)

iii) Use the results in (a)-(b) above to find the solution of the system. (3 marks)

QUESTION FOUR (20 MARKS)

Taking $x_i = 0,\ i = 1,2,3;$ as the initial approximation, solve the following system of equations:

$$4x_1 + x_2 + x_3 = 2,$$
$$x_1 + 5x_2 + 2x_3 = -6,$$
$$x_1 + 2x_2 + 3x_3 = -4.$$

a) Using Jacobi iteration method. (10 marks)

b) Using Gauss-Seidel iteration method. (10 marks)

Restriction: Perform five iterations in each case.

QUESTION FIVE (20 MARKS)

Develop the connection between rational functions and continued fractions in the case,

$$y(x) = \frac{a_0 + a_1x + a_2x^2}{b_0 + b_1x + b_2x^2}.$$

(20 marks)

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