UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 541: FLUID DYNAMICS I

DATE: APRIL 9, 2018 **TIME: 2:00 PM – 5:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) For a certain two-dimensional flow field, the velocity is given by the equation

$$\vec{V} = 4xy\vec{\imath} + 2(x^2 - y^2)\vec{\jmath}.$$

Is this flow irrotational? Justify. (5 marks)

b) The velocity components for a certain incompressible, steady flow field are

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z$$
$$w = ?$$

Determine the form of z components, w, required to satisfy the continuity equation. (6 marks)

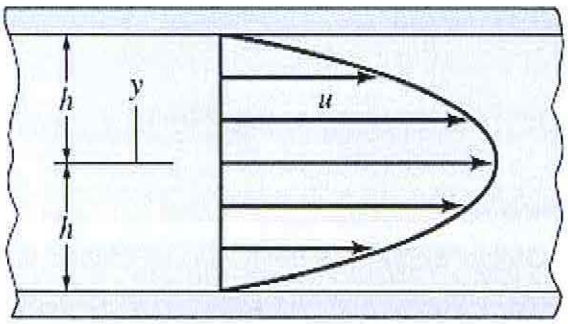
c) A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the Reynold's number, Re, defined as $\frac{\rho VD}{\mu}$, where ρ is the fluid density, V the mean fluid velocity, D the pipe diameter, and μ the fluid viscosity. A Newtonian fluid having a viscosity of $0.38 Ns/m^2$ and a specific gravity of 0.91 flows through a 25-mm diameter pipe with a velocity of $2.6m/s$. Determine the value of the Reynold's number. (4 marks)

d) A flow field is described by the equation $\psi = y - x^2$.

i) Derive an expression for the velocity V at any point in the flow field. (3 marks)

ii) Calculate the vorticity. (2 marks)

e) The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (as shown in the figure below)



is given by the equation,

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right],$$

where V is the mean velocity.

The fluid has a viscosity of $0.04 lb.s/ft^2$. When $V = 2ft/s$ and $h = 0.2in$, determine the shearing stress acting on the bottom of the wall. (4 marks)

f) The Cartesian coordinates x, y, and z, components of the continuity equation are;

$$\oiiint \frac{\partial(\rho u)}{\partial t} dV + \oiint (\rho\vec{V}.dS)u = -\oiiint \frac{\partial p}{\partial x} dV + \oiiint \rho f_x \, dV + \oiiint (F_x)_{viscous} \, dV;$$

$$\oiiint \frac{\partial(\rho v)}{\partial t} dV + \oiint (\rho\vec{V}.dS)v = -\oiiint \frac{\partial p}{\partial y} dV + \oiiint \rho f_y \, dV + \oiiint (F_y)_{viscous} \, dV;$$

$$\oiiint \frac{\partial(\rho w)}{\partial t} dV + \oiint (\rho\vec{V}.dS)w = -\oiiint \frac{\partial p}{\partial z} dV + \oiiint \rho f_z \, dV + \oiiint (F_z)_{viscous} \, dV;$$

where u, v, and w are velocity components along x, y, and z, respectively; ρ is the density of the fluid; p is the pressure; $\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$; f_x, f_y, and f_z, are body forces along the respective axes; and $(F_x)_{viscous}$, $(F_y)_{viscous}$, and $(F_z)_{viscous}$, are viscous terms along the respective axes.

Using the divergence or Gauss's theorem, convert the surface integral into a volume integral, and consequently find Navier-Stokes equations. (6 marks)

QUESTION TWO (20 MARKS)

For an inviscid flow without body forces, momentum conservation equations of fluid mechanics can be represented as

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x},$$
$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y},$$
$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z},$$

where,

$$\frac{D()}{Dt} = \frac{\partial()}{\partial t} + (\vec{V}\cdot\vec{\nabla})(),$$

is the material derivative. In the above,

$$\vec{\nabla} = \frac{\partial()}{\partial x}\vec{i} + \frac{\partial()}{\partial y}\vec{j} + \frac{\partial()}{\partial z}\vec{k}$$

while u, v, w, p, ρ, and $\vec{V}$ are as defined in Question 1(f).

Use this information to justify the Bernoulli equation's representation: (20 marks)

$$p + \frac{1}{2}\rho V^2 = \text{constant}.$$

QUESTION THREE (20 MARKS)

a) Continuity equation relates flow field variables at a point of the flow in terms of the fluid density and the fluid velocity vector:

$$\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot\vec{V} = 0,$$

where the variables are as defined in Question 2 above. Show how this important equation is obtained. (8 marks)

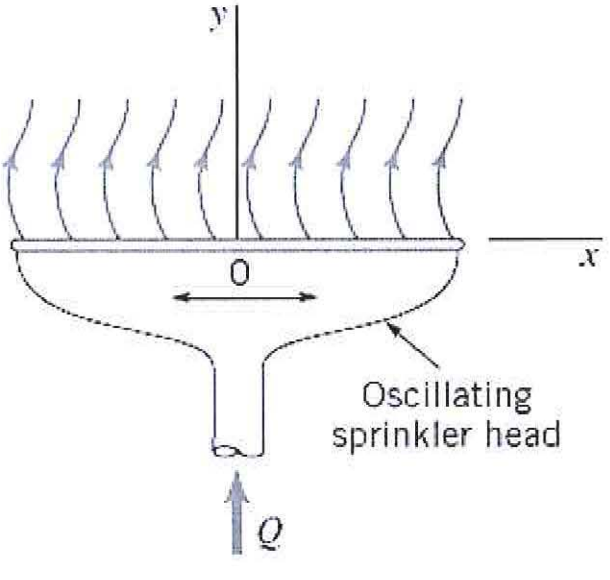
b) In a two-dimensional incompressible flow, the fluid velocity components are given by $u = x - 4y$ and $v = -y - 4x$.

i) Show that the flow satisfies the continuity equation. (2 marks)

ii) Obtain the expression for the stream function. (4 marks)

iii) If the flow is irrotational, obtain the expression for the velocity potential. (6 marks)

QUESTION FOUR (20 MARKS)



Water flowing from the oscillating slit shown in the above figure produces a velocity field given by

$$V = u_o \sin\left[\omega\left(t - \frac{y}{v_0}\right)\right]\vec{\imath} + v_0\vec{\jmath},$$

where u_0, v_0, and w are constants.

Thus the y component of velocity remains constant ($v = v_0$) and the x component of velocity at $y = 0$ coincides with the velocity of the oscillating sprinkler head [$u = u_0 \sin(\omega t)$ at $y = 0$].

a) Determine the streamline that passes through the origin at $t = 0$; at $t = \frac{\pi}{2\omega}$.

(6 marks)

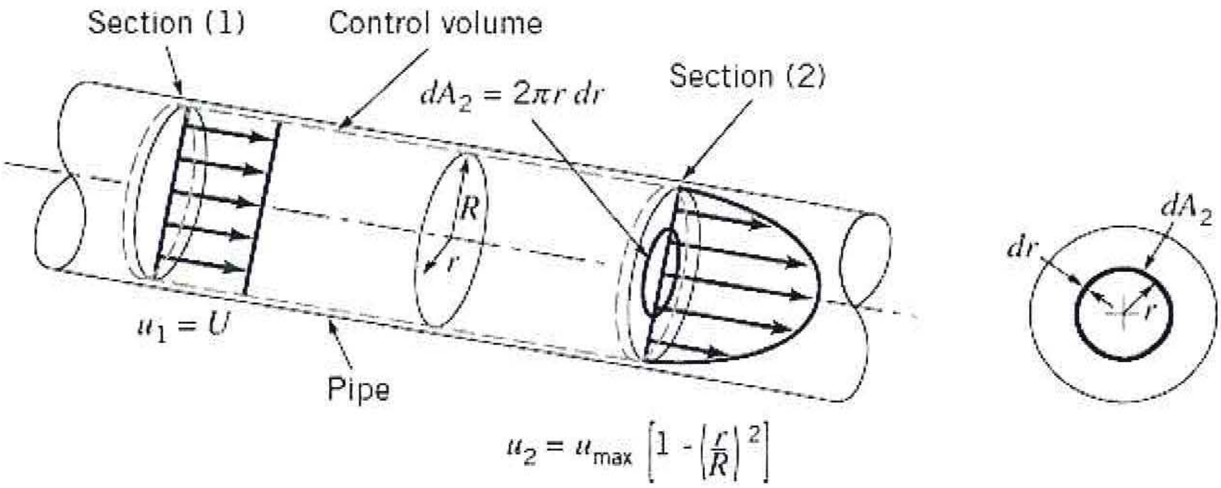
b) Determine the pathline of the particle that was at the origin at $t = 0$; at $t = \frac{\pi}{2\omega}$.

(10 marks)

c) Discuss the shape of the streakline that passes through the origin. (4 marks)

QUESTION FIVE (20 MARKS)

Incompressible, laminar water flow develops in a straight pipe having radius R as indicated in the figure below.



At section (1), the velocity profile is uniform; the velocity is equal to a constant value U and is parallel to the pipe axis everywhere.

At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall, and a maximum value of u_{max} at the centerline.

a) How are U and u_{max} related? (8 marks)

b) If the flow is vertically upward, develop an expression for the fluid pressure drop that occurs between sections (1) and (2). (12 marks)

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