UNIVERSITY OF EMBU

## 2017/2018 ACADEMIC YEAR

## TRIMESTER EXAMINATIONS

## FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

## SMA 641: NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

**DATE: AUGUST 3, 2018** **TIME: 2:00 PM – 5:00 PM**

**INSTRUCTIONS:**

**Answer Question ONE and ANY other two Questions**

**QUESTION ONE (30 MARKS)**

a) Discuss consistency, convergence and stability of a numerical method. (3 marks)

b) Explain the advantages and shortcoming of Finite difference method (FDM) over Finite volume method (FVM) and Finite Element Method (FEM) (3 marks)

c) Consider an elliptic partial differential equation $\nabla^3 u = f(x, y, z)$where $\{x, y, z\} \varepsilon P$ and

$a \leq x \leq b,\ c \leq y \leq d$ and $e \leq z \leq f$.

i) Evaluate central difference for $\nabla^3 u$ using Taylor series (3 marks)

ii) Determine the order and expression of local truncation error in (i) above. (2 marks)

iii) Determine the finite difference scheme of $\nabla^3 u = f(x, y, z)$ (3 marks)

d) Discretize the parabolic partial differential equation below using forward difference method

$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2},$

i) Evaluate central difference for $\frac{\partial^2 u}{\partial x^2}$ and forward difference for $\frac{\partial u}{\partial t}$ using Taylor series (3 marks)

ii) Determine the forward difference scheme $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ (3 marks)

iii) Determine the order and expression of local truncation error in (i) above (2 marks)

e) Discretize the parabolic partial differential equation below using backward difference

$$\frac{\partial u}{\partial t} = \alpha^2 \nabla^2 u$$

i) Evaluate central difference for $\nabla^2 u$ and backward difference for $\frac{\partial u}{\partial t}$ using Taylor series (3 marks)

ii) Determine the forward difference scheme $\frac{\partial u}{\partial t} = \alpha^2 \nabla^2 u$ (3 marks)

iii) Determine the order and expression of local truncation error in (i) above (2 marks)

## QUESTION TWO (20 MARKS)

A thin plate in the shape of a square is held at $0^0$c on two adjacent boundaries while the heat on the other boundaries increases linearly from $0^0$c on one corner to $120^0$c where the sides meet. Assuming the temperature distribution is governed by Laplace equation for $\{x, y\} \varepsilon R$

$0 \le x \le 1, 0 \le y \le 1$ with boundary conditions $u(0, y) = u(x, 0) = 0, u(x, 1) = 120x$ *and* $u(1, y) = 120y$. If $h = k = \frac{1}{3}$. Determine finite difference equations at the interior nodes and approximate the temperature at each node

## QUESTION THREE (20 MARKS)

Consider a heat equation $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad 0 < x < l, t > 0$ with boundary conditions $u(0, t) = u(l, t) = 0 \quad t > 0$ and initial conditions $u(x, 0) = Sin(\pi x) \quad 0 \le x \le 1$. Use step sizes along x and t to be 0.1 and 0.01 respectively. Approximate up to t=0.5 using forward difference method. If the analytical solution is $u(x,t) = e^{-\pi^2 t}\text{Sin}(\pi x)$, comment about its convergence and consistency

**QUESTION FOUR (20 MARKS)**

a) Explain the disadvantages of Newton Forward difference and Newton backward difference (2 marks)

b) Discretize the parabolic partial differential equation below using Crank-Nicolson method starting from Taylors series. Determine the error term and its order.

$$\frac{\partial w}{\partial t} = \theta^2 \nabla^3 w$$ (18 marks)

**QUESTION FIVE (20 MARKS)**

Consider the Hyperbolic problem $\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = 0, 0 < x < 1,\ t > 0$ with boundary conditions $u(0,t) = u(1,t) = 0, t > 0$ and initial conditions $u(x,0) = \text{Sin}(\pi x), 0 \leq x \leq 1$ and $\frac{\partial u(x,0)}{\partial t} = 0, 0 \leq x \leq 1)$. Solve the problem using finite difference method using step size on x-axis to be 0.1 and y-axis to be 0.05. If the analytical solution is $u(x,t) = \sin(\pi x)\cos(2\pi t)$, determine the absolute errors at mesh point.

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