UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

TRIMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 642: TOPICS IN NUMERICAL ANALYSIS

DATE: AUGUST 6, 2018 **TIME: 2:00 PM – 5:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Explain factors considered when choosing a discretization method (3 marks)

b) Determine the analytical solution of $-\frac{d^2u}{dx^2} - u = -x^2,\ 0 < x < 1$ and $u(0) = u(1) = 0$

(5 marks)

c) Explain an integral wavelet transform and continuous wavelet transform of a function $f(x) \varepsilon L^2$ to some mother wavelet ψ and also (2 marks)

d) Explain Inverse wavelet transforms. (2 marks)

e) Compare Finite Element Method and Finite difference Method as discretization methods. (4 marks)

f) Use a diagram of a spring to explain stiffness (5 marks)

g) Explain the concept of element based thinking in finite element (5 marks)

h) Explain steps involved in solving a problem using spectral method (4 marks)

QUESTION TWO (20 MARKS)

Solve the following problem using Finite element method and compare the solution with 1(b)

$$-\frac{d^2u}{dx^2} - u = -x^2, \qquad 0 < x < 1$$
$$u(0) = u(1) = 0$$

QUESTION THREE (20 MARKS)

Consider a one-dimensional wave-like equation $u_{tt} - \frac{x^2}{2}u_{xx} = 0, 0 < x < 1$ and $t > 1$ subject to initial conditions $u(x, 0) = x, \dot{u}(x, 0) = x^2$ and boundary conditions

$u(0, t) = 0, u(1, t) = 1 + \text{Sinh}(t), t > 0$. Use resolution level J=3 of Haar wavelets to solve one dimensional wave-like equation.

QUESTION FOUR (20 MARKS)

Consider a domain $\int_\Omega \chi_n(x)Rdx = 0, \qquad n = 0, \ldots\ldots\ldots, N.$

a) Name three mostly common spectral methods (3 marks)

b) Use the domain above to explain steps involved in solving a problem using (i) above (17 marks)

QUESTION FIVE (20 MARKS)

Derive the solution of the equation below using spectral method

$u_t + c(x)u(x) = 0, \qquad 0 \leq x \leq 2\pi$ and $0 \leq t \leq 9$

$u(x, 0) = \exp(-100(x-1)^2)$

$c(x) = \frac{1}{5} + \sin^2(x-1)$

$u(0, t) = u(2\pi, t)$ periodic boundary condition

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