UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

TRIMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 643: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: AUGUST 8, 2018 **TIME: 2:00 PM – 5:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions. Exam Duration is 3 hours.

QUESTION ONE (30 MARKS)

a) Using relevant expressions describe a classical Euler-Poisson-Darboux (EPD) equation in general form (5 marks)

b) Using relevant expressions explain explicitly Rayleigh–Ritz approximation (5 marks)

c) Explain the fundamental properties of a well posed Partial differential equation. (5 marks)

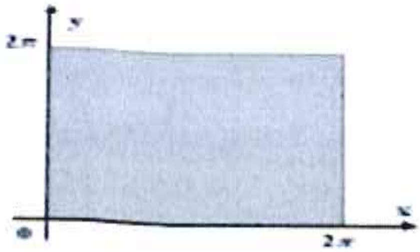
d) Let f(x) be a function such that $f(0) = f(3) = 0, \int_0^3 [f(x)]^2 dx = 1,$ and $\int_0^3 [f'(x)]^2 dx = 1$. Find such a function if you can. If it cannot be found explain why not. (5 marks)

e) Determine the eigenvalues λ_n and eigenfunctions X_n of the following $X'' + \lambda X = 0$,

i) with Boundary conditions $X(0) = X(l) = 0$ (3 marks)

ii) with Boundary conditions $X(0) = X'(l) = 0$ (2 marks)

f) Consider the heat equation $u_t = u_{xx} + u_{yy}$ on the square $\Omega = \{0 \le x \le 2\pi, 0 \le y \le 2\pi\}$ with periodic boundary conditions and with initial data $u(0,x,y) = f(x,y)$. Find the solution using separation of variables. (5 marks)



QUESTION TWO (20 MARKS)

Consider the eigenvalue problem $-\Delta u(x) = \lambda u(x), x\varepsilon\Omega$ with the more general conditions

$$u(x) = 0, x\varepsilon C_1 \subset d\Omega, \qquad \frac{\partial u}{\partial n}(x) + \alpha(x)u(x) = 0, x\varepsilon C_2 \subset d\Omega$$

where C_1 and C_2 are disjoint subset of $d\Omega$ with $C_1 \cap C_2 = d\Omega$. Solve the problem using finite difference method

QUESTION THREE (20 MARKS)

a) State and prove the Minimum Principle for the First Eigenvalue (10 marks)

b) Suppose $u \in C^2(R^n \times (0,\propto))$ solves

$u_{tt} = c^2 \Delta u \quad x\epsilon R^n, t > 0, u(x,0) = g(x),$

$u_t(x,0) = h(x)\ x\epsilon R^n$, where g and h have compact support

i) Define an energy function for U(x,t) (2 marks)

ii) Show that energy function E(t) is constant and solution is unique

(8 marks)

QUESTION FOUR (20 MARKS)

a) State and prove Minimum Principle for the nth Eigenvalue (10 marks)

b) Consider a wave equation $\frac{1}{c^2(x)}u_{tt} = \Delta u \quad x\epsilon\Omega, \ \frac{\partial u}{\partial t} - \alpha(x)\frac{\partial u}{\partial t} = 0$ on $\partial\Omega \times R$. Assume that $\alpha(x)$ is of one sign for all x(i.e α always positive or negative). For the energy $E(t) = \frac{1}{2}\int_\Omega \frac{1}{c^2(x)}u_t^2 + |\nabla u|^2 dx$ Show that the sign $\frac{dE}{dt}$ is determined by the sign of α.

(10 marks)

QUESTION FIVE (20 MARKS)

For the eigenvalue problem $-u'' = \lambda u$ in the interval $(0, 1)$, with $u(0) = u(1) = 0$, choose the pair of trial functions $x - x^2$ and $x^2 - x^3$ and compute the Rayleigh–Ritz approximations to the first two eigenvalues. Compare with the exact values.

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