

**W1-2-60-1-6**

**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

# **UNIVERSITY EXAMINATIONS 2015/2016**

**SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF ……………………………………………..**

**SMA 2202 : ALGEBRAIC STRUCTURES**

**DATE: AUGUST 2015 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

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**SECTION ONE**

**QUESTION ONE [30 MARKS]**

1. In a set (set of positive integers) we define a binary operation by

a\*b = min[ a, b] a,b. Show that \* is

1. Commutative [2 marks]
2. Associative [2 marks]
3. Prove that an identity element in a binary structure (S,A) is unique. [5 marks]
4. Let Mmin(ℝ) demote the set of all Mxn matrices whose entries are real-endowed with addition of matrices. Show that Mmin(ℝ) is a group. [8 marks]
5. (i) In Klein 4-group, find all the subgroups and draw the subgroup diagram. [4 marks]

(ii) Find all the subgroups of the group (ℤ4, +) and draw the subgroup diagram. [4 marks]

1. Show that 3 and 10 are relatively prime. [3 marks]
2. Define an Isomorphic map from a group G to a group G1. [2 marks]

**QUESTION TWO [20 MARKS]**

1. Define acyclic group. [2 marks]
2. Show that the solution of is a cyclic group [4 marks]
3. Find the generators of  [2 marks]
4. Prove that every cyclic group is a belian. [4 marks]
5. For integers between 1 and 10, state 5 parts of integers which are relatively prime. [5 marks]
6. Show that 3 and 10 are relatively prime. [3 marks]

**QUESTION THREE [20 MARKS]**

1. State Lagrange theorem. [2 marks]
2. Find all left cosets of 3ℤ in ℤ. [8 marks]
3. Find all the right and left cosets of the subgroup  of D4 and draw the table for the right and left quotient groups of H in D4. [10 marks]

**QUESTION FOUR [20 MARKS]**

1. If R is aring with additive identity 0, show that for any we have
2. 0a=a0=0 [3 marks]
3. A(-b)=(-a)b =-(ab) [3 marks]
4. (-a)(-b) =ab. [3 marks]
5. Show that the groups <ℤ, +> and <2ℤ, +> are isomorphic under the map  with  [3 marks]
6. Show that the rings 2ℤ and 3ℤ are not isomorphic. [3 marks]
7. Describe all units in each of the following rings.
8. Q [2 marks]
9. Z+Z [2 marks]