

W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**University Examinations 2015/2016**

**SECOND YEAR FIRST SEMESTER EXAMINATION BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE**

**SMA 2202: ALGEBRAIC STRUCTURES**

**DATE AUGUST 2015 (2HOURS)**

**INSTRUCTIONS ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30MARKS)**

1. Determine whether the rule  is operation on the set of integers (3marks)
2. Consider the operation on$ R$. Determine whether;
3. \* is commutative (2marks)
4. \* is associative (2marks)
5. $R$ is an identity element with respect to \* (3marks)
6. Every $x\in R$ has an inverse with respect to \* (3marks)
7. Prove that x cannot have more than one inverse in a semigroup (4marks)
8. Let $S=N×N where N$ is the set of natural numbers. Let $δ$ be the operation on S defined by $\left(a,b\right)δ\left(a^{'},b^{'}\right)=\left(a+a^{'},b+b^{'}\right).$ Show that $\left(S,δ\right)$ is a semi group

 (4marks)

1. Let $U\_{16}$ denote a residue system modulo 16 system consisting of integers relatively prime to 16. Work out the multiplication table for $U\_{16}$ and then conclude if $U\_{16}$a group is. (6marks)
2. List all the zero divisors of $Z\_{14}$ (3 marks)

**QUESTION TWO (20MARKS)**

1. Proof that every cycle group is abelian. Hence or otherwise, show if the converse holds (6marks)
2. Define a ring and show that $Z\_{5}$ the set of integers mod 5 is both a ring and a field.

 (8marks)

1. Let $A=\left\{1,2,3\right\}.$ Let  be permutations of$S\_{3}$. Compute all the left cosets of the subgroup $H=\{l\_{0,}μ\_{1}$}

(6marks)

**QUESTION THREE (20MARKS)**

a)

i) Define the term dihedral group, $D\_{3}$ (2marks)

ii) Hence or otherwise, make the Cayley table for $D\_{3}$ (10marks)

b) Let be an operation defined on $Z$ such that for all $, b\in Z, a∅b=(ab)^{a}$ . Show if is associative, commutative? (5marks)

c) Consider the set $Z$ of natural numbers, and let $\*$ denote the operation of the least common multiples (1cm) on$ Z$. Is$ (Z,\*$) a semigroup? Is it commutative? (2marks)

**QUESTION FOUR (20MARKS)**

1. Consider the group $G=\{8,3,9,2,5,7,4,6,1,10\}$ under multiplication modulo 11
2. Work out the multiplication table of G (4marks)
3. Evaluate  (4 marks)
4. State without proof the langrage’s theorem (2marks)
5. Let G be a group
6. What does it mean to say that G is a group? (3marks)
7. Proof that  is a binary operation on G, then the left and right cancellation laws hold in G. (7marks)