W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATION 2018/2019

EXAMINATION FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE/ FINANCIAL ENGINEERING/ STATISTICS/ BIOSTATISTICS

STA 2306: REAL ANALYSIS FOR STATISTICS

DATE: DECEMBER 2018 **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE COMPULSORY (30MARKS)

a. By ratio test determine whether the following series converge $\sum_{n=1}^{\infty} \frac{(2n)!}{7^n (n!)^2}$ [5marks]

b. Given that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, use the limit comparison test to determine the convergence of $\sum_{n=1}^{\infty} \sin\frac{1}{n}$ [4marks]

c. If $a_n \nrightarrow 0$ as $n \longrightarrow 0$ then $\sum_{n=1}^{\infty} a_n$ diverges. Prove [4marks]

d. Evaluate $\lim\limits_{(x,y)\to(0,0)} \left[\frac{x^3 y}{x^6+y^2}\right]$ [5marks]

e. Find all the second order derivatives for $f(x,y) = \cos(2x) - x^2 e^{5y} + 3y^2$ [5marks]

f. State Clairaut's theorem [2marks]

g. Find the Fourier series expansion for $f(x) = x^2$ [5marks]

$f(x) = \frac{\pi^2}{3} - 4\pi \cos x + \frac{4\pi}{2^2} \cos 2x - \frac{4\pi}{3^2} \cos 3x + \ldots$

$a_0 = \frac{\pi^2}{3}$

$a_n = \begin{cases} -4\pi/n^2 & \text{when odd} \\ \frac{4\pi}{n^2} & \text{when even.} \end{cases}$

QUESTION TWO (20 MARKS)

$b_n = 0.$

a. Use the Fourier series for x^2 to show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ [4marks]

b. Let $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ be series of real numbers such that $\frac{a_n}{b_n}$ tends to a finite non zero limit as $n \to \infty$. Prove that either series both converge or diverge [10marks]

c. Show that the following series converges

   i. $\sum_{n=1}^{\infty} \frac{n^3 - 1}{4n^5 - 3n^2 + 3}$ [3marks]

   ii. $\sum_{n=1}^{\infty} \left[\frac{n+1}{n^2+1}\right]^2$ [3marks]

$\frac{\pi^2}{3} - 4\pi \cos x + \frac{4\pi}{2^2} \cos 2x - 4/3^2 \cos 3x$

$\sum_{n=1}^{\infty} 1/n^2 = 1/1^2 + 1/2^2 + 1/3^2 + \ldots$

let $x = \pi$

$\pi^2 = \pi^2$

QUESTION THREE (20 MARKS)

a. The total weekly profit (in dollars) that Acrosonic company realized in producing and selling its bookshelf loudspeaker systems is given by the profit function

$$P(x, y) = -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 120x + 100y - 5000$$

Where x denotes the number of fully assembled units and y the number of kits produced and sold per week. The management decides that production of the loudspeaker systems should be restricted to a total of exactly 230 units per week. Under this condition, how many fully assembled units and how many kits should be produced per week to maximize Acrosonic's weekly profit? [10marks]

b. Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{n(x-2)^n}{n+2}$ [5marks]

c. State Wierstrass M-Test hence apply it to Evaluate $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ [5marks]

QUESTION FOUR (20 MARKS)

a. Evaluate if Clairaut's theorem holds for $z = e^{x^2+y^2} \tan \sqrt{x}$ [8marks]

b. Apply integral test on $\sum_{n=1}^{\infty} \frac{1}{n^2} (\log n)^3 = -3$ [7marks]

c. Determine the point on the plane $4x - 2y + z = 1$ that is closest to the point $(-2, -1, 5)$ [5marks]