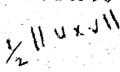
SMA 2201: LINEAR ALGEBRA I





JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR EXAMINATION FOR BACHELOR OF SCIENCE IN ACTURIAL SCIENCE, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN BIOSTASTISTIC & BACHELOR OF RENEWABLE ENERGY AND GEOPHYICS.

SMA 2201: LINEAR ALGEBRA I

DATE: JANUARY 2018

TIME: 2 HOURS

INSTRUCTIONS: Answer question ONE and any other TWO questions

OUESTION ONE (COMPULSORY)

a)

i. Find scalars x, y and z if $x(5\hat{i} + \hat{j}) + y(\hat{j} - k) + z\hat{k} = 5\hat{i} + 3\hat{j} + k$. (2 marks)

ii. Let u = (2,2,3), v = (-1,-2,6) and k = (2,2,-4). Find:

a. The vector that is orthogonal to both u and v.

(2 marks)

b. The angle between v and k.

(2 marks)

iii. Find the area of the triangle determined by the vectors u = (1, -1, 2) and v = (0, 3, 1).

(3 marks)

b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ to reduced row echelon form.

(4 marks)

c) Calculate the determinant of $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix}$ and state whether A is linearly independent or linearly dependent.

d) Define linear combination vectors. Hence determine whether the vector (0, 1, 0) is a linear combination of the vectors (3,1,-9), (2,5,4) and (-1, -2, -3). (5 marks)

e) Determine whether all the vectors of the form (a, a, 0) is a subspace of \mathbb{R}^3 . (3 marks)

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- 1) Prove the triangular inequality $||u+v|| \le ||u|| + ||v||$ where u and v are in \mathbb{R}^3 (2 marks) (2 marks)
- g) Define rank and nullity as used in linear functions
- h) Find the distance between the point (1, -4, -3) and the plane 2x 3y + 6z = -1. (2 marks)

QUESTION TWO (20 marks)

- a) Determine the equation of the plane passing through the points A(4,1,1), B(0,0,5) and (5 marks) C(2,7,2).
- b) Find the parametric and symmetric equation of a line passing through the points (3 marks) P(3,-1,2) and parallel to the vector (2,1,3)
- (2 marks) c) Calculate 4
- d) Solve the following system by Gaussian Elimination and back substitution.

$$x + 2y + 3z = 4$$

 $2x - 4y + 2z = 2$
 $-x + + z = 2$ (5 marks)

-x + +z = 2e) Reduce the following matrix to reduced row echelon form $\begin{pmatrix} 1 & 0 & 1 & z \\ 1 & 4 & -3 & -2 \\ 1 & -1 & 2 & 3 \\ 0 & 3 & -2 & 1 \end{pmatrix}$. (5 marks)

QUESTION THREE (20 marks)

- a) i. Define the Spanning set or generators of a vector space V.
 - (2 marks)
 - ii. Show that vectors $e_1 = (1,0,0)$, $e_2 = (0,1,0)$ and $e_3 = (0,0,1)$ span the vector space \mathbb{R}^3 . (3 marks)
- Consider the set $S = \{(1,0,1), (2,2,0), (3,3,3)\}$
 - i. Determine whether the set S spans \mathbb{R}^3 . (3 marks) ~
 - ii. Determine whether the set S is linearly independent. (2 marks)iii. Is S a basis for \mathbb{R}^3 ? (1 mark)
- c) Consider the linear function $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by f(x,y) = (x-2y,2x-4y).
 - i. Find a basis for the kernel of f and the nullity of f. (3 marks)
 - ii. Find a basis for the range of f and the rank of f. (3 marks)
- d) Let $v \in \mathbb{R}^n$ and k be any scalar. Show that ||kv|| = |k|||v||(3 marks)

OUESTION FOUR (20 marks)

a) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be a function defined by f(x, y, z) = (x + y, y + z, x - z). Show that f is a linear transformation. Find the matrix representation of f with respect to the standard basis for \mathbb{R}^3 . (6 marks)



- b) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be a function defined by f(x, y, z) = (x + y, y + z, x z). Find the rank and nullity of f. (5 marks)
- c) Given that $B = \{(x, y) \in \mathbb{R}^2 | x = 2y \}$, determine whether B is a subspace of \mathbb{R}^2 .

 (3 marks)
- d) Determine whether the vector (3, 3, 3) is a linear combination of the vectors w = (1, -2, 2) and v = (0, 3, -1).
- e) Let $V = \mathbb{R}$ define addition and scalar multiplication by $a \oplus b = 3a + 3b$ and $k \odot a = ka$. Show that this addition is commutative but not associative. (3 marks)