$\frac{1}{2}\|u \times v\|$

JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR EXAMINATION FOR BACHELOR OF SCIENCE IN ACTURIAL SCIENCE, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN BIOSTASTISTIC & BACHELOR OF RENEWABLE ENERGY AND GEOPHYICS.

SMA 2201: LINEAR ALGEBRA I

DATE: JANUARY 2018 **TIME: 2 HOURS**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY)

a)

i. Find scalars x, y *and* z if $x(5\hat{\imath} + \hat{\jmath}) + y(\hat{\jmath} - \hat{k}) + z\hat{k} = 5\hat{\imath} + 3\hat{\jmath} + k$. (2 marks)

ii. Let $u = (2,2,3), v = (-1,-2,6)$ and $k = (2,2,-4)$. Find:

a. The vector that is orthogonal to both u and v. (2 marks)

b. The angle between v and k. (2 marks)

iii. Find the area of the triangle determined by the vectors $u = (1,-1,2)$ and $v = (0,3,1)$.

(3 marks)

b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ to reduced row echelon form. (4 marks)

c) Calculate the determinant of $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix}$ and state whether A is linearly independent or linearly dependent. (3 marks)

d) Define linear combination vectors. Hence determine whether the vector $(0,1,0)$ is a linear combination of the vectors $(3,1,-9), (2,5,4)$ and (-1, -2, -3). (5 marks)

e) Determine whether all the vectors of the form $(a, a, 0)$ is a subspace of $\mathbf{R}^3$.(3 marks)

f) Prove the triangular inequality $||u + v|| \leq ||u|| + ||v||$ where u and v are in $\mathbb{R}^3$ (2 marks)

g) Define rank and nullity as used in linear functions (2 marks)

h) Find the distance between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$. (2 marks)

QUESTION TWO (20 marks)

a) Determine the equation of the plane passing through the points $A\ (4,1,1), B\ (0,0,5)$ and $C\ (2,7,2)$. (5 marks)

b) Find the parametric and symmetric equation of a line passing through the points $P(3, -1, 2)$ and parallel to the vector $(2, 1, 3)$ (3 marks)

c) Calculate $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$. (2 marks)

d) Solve the following system by Gaussian Elimination and back substitution.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x - 4y + 2z &= 2 \\ -x + \quad + z &= 2 \end{aligned}$$

(5 marks)

e) Reduce the following matrix to reduced row echelon form $\begin{pmatrix} 1 & 0 & 1 & 2 \\ 1 & 4 & -3 & -2 \\ 1 & -1 & 2 & 3 \\ 0 & 3 & -2 & 1 \end{pmatrix}$. (5 marks)

QUESTION THREE (20 marks)

a)
   i. Define the Spanning set or generators of a vector space V. (2 marks)
   ii. Show that vectors $e_1 = (1,0,0), e_2 = (0,1,0)$ *and* $e_3 = (0,0,1)$ span the vector space $\mathbb{R}^3$. (3 marks)

b) Consider the set $S = \{(1, 0, 1), (2, 2, 0), (3, 3, 3)\}$
   i. Determine whether the set S spans $\mathbb{R}^3$. (3 marks)
   ii. Determine whether the set S is linearly independent. (2 marks)
   iii. Is S a basis for $\mathbb{R}^3$? (1 mark)

c) Consider the linear function $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $f(x, y) = (x - 2y, 2x - 4y)$.
   i. Find a basis for the kernel of f and the nullity of f. (3 marks)
   ii. Find a basis for the range of f and the rank of f. (3 marks)

d) Let $v \in \mathbb{R}^n$ and k be any scalar. Show that $||kv|| = |k|||v||$ (3 marks)

QUESTION FOUR (20 marks)

a) Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a function defined by $f(x, y, z) = (x + y, y + z, x - z)$. Show that f is a linear transformation. Find the matrix representation of f with respect to the standard basis for $\mathbb{R}^3$. (6 marks)

b) Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a function defined by $f(x, y, z) = (x + y, y + z, x - z)$. Find the rank and nullity of f. (5 marks)

c) Given that $B = \{(x, y) \in \mathbb{R}^2 | x = 2y\}$, determine whether B is a subspace of $\mathbb{R}^2$. (3 marks)

d) Determine whether the vector $(3, 3, 3)$ is a linear combination of the vectors $w = (1, -2, 2)$ and $v = (0, 3, -1)$. (3 marks)

e) Let $V = \mathbb{R}$ define addition and scalar multiplication by $a \oplus b = 3a + 3b$ and $k \odot a = ka$. Show that this addition is commutative but not associative. (3 marks)