W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

STA 2291 Actuarial Mathematics I

DATE: January 2018   TIME: 2 HOURS

Instructions: Answer QUESTION ONE and any other TWO QUESTIONS.

QUESTION ONE (30 MARKS) (COMPULSORY)

(a) Calculate

   (ii) $_4q_{45}$ (1 mark)

   (iii) $_{4|2}q_{[42]}$ (2 marks)

Basis: AM92 Mortality

(b) Calculate the probability of survival to age 60 exact using ELT15 (Males) for a life aged 45.5 exact using two approximate methods. State any assumptions you make.

                                                (6 marks)

(c) Using the AM92 ultimate mortality and an interest rate of 4% per annum, compute the following expected present values.

   (i) $A_{42:\overline{21}}$ (3 marks)

   (ii) $\ddot{a}^{(12)}_{44:\overline{3}|}$ (4 marks)

   (iii) $\ddot{a}_{40:\overline{5}|}$ (3 marks)

(d) (i) State the age ranges over which Gompertz' law is an appropriate model for human mortality. (1 mark)

(ii) Show that, under Gompert'z law, the probability of survival from age x to age $x + t$ is equal to:

$$\left[\exp\left(-\frac{B}{\log c}\right)\right]^{c^x(c^t-1)}$$

(4 marks)

(e) A life insurance company issues a 30-year endowment assurance policy to a life aged 30 years exact. Level premiums are payable monthly in advance throughout the term of the policy. The sum assured of 500,000 is payable at maturity or immediately on death of the life insured, if occurs earlier.

Calculate the monthly premium on the following basis: Mortality: AM92 Select, Interest: 4% per annum, Initial expenses: 300 plus 60% of the gross annual premium, Renewal expense of 4% of the second and subsequent premiums. (6 marks)

QUESTION TWO (20 Marks)

(b) You are given:

- The probability that (30) survives to reach age 50 is 0.75.
- The probability that (30) dies between the ages of 50 and 65 is 0.15.
- $l_{30} = 1000$

Calculate l_{65}. (3 marks)

(b) A survival model is being constructed which is based on the assumption that there is a uniform distribution of deaths between integral ages.

(i) Write down an expression for l_{x+t} in terms of l_x and d_x for $0 \le t \le 1$. (1 marks)

(ii) Show that $_t q_x = t \cdot q_x$. (2 marks)

(iii) Show that if $0 \le s, t \le 1$ such that $s + t \le 1$, then,

$$_t q_{x+s} = \frac{t \cdot q_x}{1 - s \cdot q_x}$$

(3 marks)

(c) (i) Outline the difference between the methods of Unform Distribution of Deaths and Constant Force of Mortality. (2 marks)

(ii) Calculate $_{1.75}q_{45.75}$ using Constant Force of Mortality and PMA92C20 (3 marks)

(d) (i) Derive from first principles an expression for the variance of the benefits payable under an endowment assurance with benefits payable at the end of the year of death. Use a sum assured of S. (3 marks)

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(ii) An endowment assurance policy of amount 1 is sold. The benefit is payable at the end of the year of death to a life initially aged 35 exact, with a term of 10 years.

Basis

Basis: Mortality: AM92 Ultimate
Interest: 6% per annum

Calculate the expected present value of this policy. (3 marks)

QUESTION THREE (20 Marks)

(a) According to a particular mortality table

$$_tp_{20} = \left(1 - \frac{t}{20}\right) \quad \text{for} \quad t < 20, \quad \text{and}$$

$$_tp_{20} = 0, \quad \text{for} \quad t \geq 20.$$

Calculate the value of $e^{\circ}_{20\,\overline{10}|}$. (4 marks)

(b) Show that $A_x = A^1_{x:\overline{n}|} + v^n\,_np_x A_{x+n}$. (4 marks)

(c) A life insurance company issues a 35-year non-profit endowment assurance policy to a life aged 30 years exact. Level premiums are payable monthly in advance throughout the term of the policy. The sum assured of 75.000 is payable at maturity or at the end of the year of death of the life insured, if occurs earlier. Calculate the monthly premium on the following basis: Mortality : AM92 Select Interest : 4% per annum Initial expenses: 250 plus 50% of the gross annual premium, Renewal expense of 5% of the second and subsequent premiums. (8 marks)

(d) A life office is to issue a 5-year term assurance policy to a life aged 35. The sum assured is 150,000 payable immediately on death within 5 years and premiums are payable annually in advance for 5 years. Calculate the amount of the annual premium.
Premium basis: Mortality: AM92 Ultimate
Interest: 4% per annum
Expenses: none (4 marks)

QUESTION FOUR (20 Marks)

(a) Let T_x be the future lifetimes of Mr Howcome, aged x, and its curtate future lifetime be denoted by K_x. Howcome buys a whole life assurance contract with a benefit of $1 at the end of the year of death, so that the present value of the benefit is

$$Z_x = v^{K_x+1}$$

By first proving that $P(K_x = k) = \,_kp_x q_{x+k},$

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$$_n E_x = v^n\,_np_x.$$

show that the expectation and variance of Z_x are given by

$$A_x = \sum_{k=0}^{\infty} v^{k+1}{}_{k|}q_x,$$

and $\mathrm{Var}(Z_x) = {}^2A_x - (A_x)^2$, respectively. Show all your working and reasoning.

(4+3+3 marks)

(b) If $x = 30$ and the sum assured is now $50,000. Calculate the standard deviation of the present value of Mr Howcome benefit using AM92 Ultimate mortality and 6% pa interest. (4 marks)

(c) Compute the expected present value and variance of the present value of a term assurance contract of $1 payable immediately on death for a life aged exactly 40, if death occurs within 30 years. Use AM92 Ultimate mortality with 4% pa interest. (6 marks)

QUESTION FIVE (20 Marks)

(a) A membership club has a constant membership of 500. X new entrants are added each year at exact age 25. Withdrawals are either by death or by retirement. 30% of those who reach age 50 retire at that time and all others retire at age 60. Express each of the following in terms of population functions:

   (i) The number of annual entrants at age 25.

   (ii) The number of members who retire each year at age 50.

   (iii) The number of members who die each year. (6 marks)

(b) A random variable T_{xy} represents the time to failure of the joint-life status (xy). (x) is subject to a constant force of mortality of 0.02 and (y) is subject to a constant force of mortality at 0.01. (x) and (y) are independent with respect to mortality. Calculate the value of $E[T_{xy}]$. (4 marks)

(c) Using AM92 ultimate mortality, calculate

   (i) $p_{62:65}$ $\blacktriangleright$ $P_{x:\overline{n}|}$ (2 marks)

   (ii) ${}_3q_{50:50}$ (2 marks)

   (iii) $\mu^1_{60:65}$ (2 marks)

(d) Two lives subject to AM92 ultimate mortality are now aged 40 and 50. Calculate the probability that the 50 year old will die at a younger age than the 40 year old.

(4 marks)